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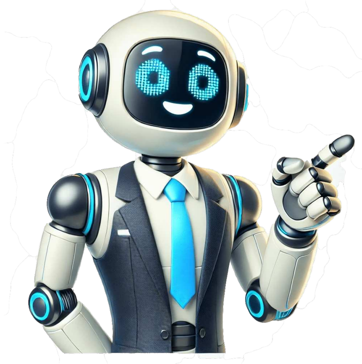

Solving quadratic equations by completing the square worksheet

Here is everything you need to know about completing the square for GCSE maths (Edexcel, AQA and OCR). You'll learn how to recognise a perfect square, complete the square on algebraic expressions, and tackle more difficult problems with the coefficient of x2 ≠ 1. You will also learn how to solve quadratic equations by completing the square, and how the completed square form links to graphs of quadratic equations. Look out for completing the square worksheets and exam questions at the end. Completing the square is an alternative method to solving quadratic equations when the quadratic cannot be factorised. A quadratic expression like x2 + 4x + 4 is called a perfect square. This is because it factorises to give (x + 2)(x+ 2), which can also be written as (x + 2)2. We can see this idea diagrammatically as follows: Most quadratic expressions are not perfect squares, and cannot be written in this form as a single squared bracket. When we complete the square, we try to fit the expression to the closest possible perfect square, with a little bit added or subtracted to make things work. Some expressions will have an 'extra' amount over from a perfect square, such as: So we would write this expression in completed square form as: Some expressions will be 'missing' an amount to make a perfect square, such as: So we would write this expression in completed square form as: While it's easy to see this using diagrams for quadratic expressions with small coefficients, we need a better method for expressions with larger coefficients. You may have noticed already that we divide the coefficient of x by 2 in order to work out the nearest perfect square. Completing the square is a really useful method for solving quadratic equations; the quadratic formula for solving quadratic equations is based on it and can be derived by completing the square. The completed square form of a quadratic expression is also really useful for identifying key points of quadratic functions, such as the maximum or minimum of a quadratic parabola (also called the vertex), without having to draw a graph. You can see this in the examples below. You may sometimes see an expression in the form (x + a) referred to as a binomial. In order to complete the square: Find the closest perfect square by dividing the coefficient of x by 2. Expand the perfect square expression. Compare the constant term in the perfect square to the original expression, and adjust as needed. Get your free completing the square worksheet of 20+ questions and answers. Includes reasoning and applied questions. DOWNLOAD FREE x Get your free completing the square worksheet of 20+ questions and answers. Includes reasoning and applied questions. DOWNLOAD FREE Complete the square for the expression Find the closest perfect square by dividing the coefficient of x by 2. The coefficient of x is 8, so when we divide this by 2, we get 4. The closest perfect square is: 2 Expand the perfect square expression. \[(x + 4)^{2} = x^{2} + 8x + 16\] 3 Compare the constant term in the perfect square to the original expression, and adjust as needed. These match (because our example was a perfect square), so we don't need to make any adjustment. The answer in complete square form is Graphically This graph shows the curve The minimum value of y occurs when the bracket equals 0. This happens when x = −4. If we substitute x = −4, we get: \[y = (-4 + 4)^{2} = 0^{2} = 0\] So the coordinates of the vertex, which is a minimum point, are (−4, 0). Complete the square for the expression Find the closest perfect square by dividing the coefficient of x by 2. The coefficient of x is 6, so when we divide this by 2, we get 3. The closest perfect square is Expand the perfect square expression. \[(x + 3)^{2} = x^{2} + 6x + 9\] Compare the constant term in the perfect square to the original expression, and adjust as needed. In order to make the constant term correct, we need to add 8, because 9 + 8 = 17. The answer in complete square form is Graphically This graph shows the curve The minimum value of y occurs when the bracket equals 0. This happens when x = −3. If we substitute x = −3, we get: \[y = (-3 + 3)^{2} + 8 = 0^{2} + 8 = 8\] So the coordinates of the vertex, which is a minimum point, are (−3, 8). As a little shortcut, the y value is just whatever number is on its own outside the bracket, and the x value is the opposite sign of the number inside the bracket. Complete the square for the expression Find the closest perfect square by dividing the coefficient of x by 2. The coefficient of x is 2, so when we divide this by 2, we get 1. The closest perfect square is Expand the perfect square expression. \[(x + 1)^{2} = x^{2} + 2x + 1\] Compare the constant term in the perfect square to the original expression, and adjust as needed. In order to make the constant term correct, we need to subtract 6, because 1 − 6 = −15. The answer in complete square form is Graphically This graph shows the curve The minimum value of y occurs when the bracket equals 0. This happens when x = −1. If we substitute x = −1, we get: \[y = (-1 + 1)^{2} – 6 = 0^{2} – 6 = -6\] So the coordinates of the vertex, which is a minimum point, are (−1, −6). This is really straightforward – just remember that your perfect square bracket will need subtraction rather than addition in the middle. Complete the square for the expression Find the closest perfect square by dividing the coefficient of x by 2. The coefficient of x is −10, so when we divide this by 2, we get −5. The closest perfect square is Expand the perfect square expression. \[(x – 5)^{2} = x^{2} – 10x + 25\] Compare the constant term in the perfect square to the original expression, and adjust as needed. In order to make the constant term correct, we need to subtract 8, because 25 − 8 = 17. The answer in complete square form is Graphically This graph shows the curve The minimum value of y occurs when the bracket equals 0. This happens when x = −5. If we substitute x = 5, we get: \[y = (5 – 5)^{2} – 8 = 0^{2} – 8 = -8\] So the coordinates of the vertex, which is a minimum point, are (5, −8). Complete the square for the expression Find the closest perfect square by dividing the coefficient of xby 2. The coefficient of x is 3, so when we divide this by 2, we get It can be tempting to use decimals, but fractions are much easier, particularly as completing the square is more likely to be examined on a non-calculator paper at GCSE. The closest perfect square is \[\left(x + \frac{3}{2}\right)^{2}\] Expand the perfect square expression. \[\left(x + \frac{3}{2}\right)^{2} = x^{2} + 3 x + \frac{9}{4}\] Compare the constant term in the perfect square to the original expression, and adjust as needed. It can be useful to think of 4 as the improper fraction this makes it easier to work out the adjustment. In order to make the constant term correct, we need to add because \[\frac{9}{4} + \frac{7}{4} = \frac{16}{4} = 4\] The answer in complete square form is \[\left(x+\frac{3}{2}\right)^{2}+\frac{7}{4}\] Graphically This graph shows the curve The minimum value of y occurs when the bracket equals 0. This happens when If we substitute we get: \[y=\left(\frac{-3}{2}+\frac{3}{2}\right)^{2}+\frac{7}{4}=0^{2}+\frac{7}{4}=\frac{7}{4}\] So the coordinates of the vertex, which is a minimum point, are \[\left(\frac{-3}{2}, \frac{7}{4}\right)\] If the coefficient of x2 is not 1, we must first factorise to get an expression where the coefficient of x2 is 1. If the expression involves the term ax2, then we take out a factor of a. Factorise. Complete the square on the expression inside the brackets: find the closest perfect square by dividing the coefficient of x by 2. Expand the perfect square expression. Compare the constant term in the perfect square to the original expression, and adjust as needed. Multiply out the factorised value. Complete the square for the expression We take out the common factor of 2, so the expression becomes \[2\left(x^{2} + 4 x-5\right)\] then complete the square on the expression inside the bracket. Find the closest perfect square by dividing the coefficient of x by 2. The coefficient of x is 4, so when we divide this by 2, we get 2. The closest perfect square is Expand the perfect square expression. Compare the constant term in the perfect square to the original expression, and adjust as needed. In order to make the constant term correct, we need to subtract 9, because 4 − 9 = −5. So the expression in complete square form is Multiply out the factorised value. Don't forget to deal with the factor of 2 that we removed at the start. So the expression: \[2\left(x^{2}+4 x-5\right)\] in complete square form is: \[2\left((x+2)^{2}-9\right)\] Finally, expand out to give a final answer: Graphically This graph shows the curve The minimum value of y occurs when the bracket equals 0. This happens when x = −2. If we substitute x = −2, we get: \[y=2(x+2)^{2}-18=0^{2}-18=-18\] So the coordinates of the vertex, which is a minimum point, are (−2, −18). Note: we can also complete the square for expressions with a negative x2 coefficient in the same way, by taking out a negative factor in the first step. For example, would be written as: \[-\left(x^{2}-5 x+3\right)\] and then completed as in Example 5. Some quadratic equations can be solved by factorising, but most require either completing the square or the quadratic formula – in fact, the quadratic formula is derived from completing the square (see worksheet). In order to solve quadratic equations using complete the square: Make sure the equation is rearranged so that the right hand side equals 0 (if necessary). Complete the square on the left hand side. Rearrange and solve the resulting equation for x. Solve the quadratic equation Make sure the equation is rearranged so that the right hand side equals 0 (if necessary). RHS already equals 0 here, so no need to do anything. Complete the square on the left hand side. \[x^{2}+6x+3=(x+3)^{2}-6\] Rearrange and solve the resulting equation for x. \[\begin{aligned} (x+3)^{2}-6&=0 \\\\ (x+3)^{2}&=6 \\\\ x+3&=\pm \sqrt{6} \\\\ x&=-3 \pm \sqrt{6} \end{aligned} \] We have two solutions to the equation: \[x=-3+\sqrt{6} \\\\ x=-3-\sqrt{6} \] It is really important to remember the ± sign when square-rooting, as most quadratic equations you're asked to solve at GCSE have two solutions or roots. Sometimes, there will be one repeated root. In most cases, your answer should be left in surd form (with the square root sign in), not converted to a decimal. Solve the quadratic equation Make sure the equation is rearranged so that the right hand side equals 0 (if necessary). Rearrange by adding 5 and 2x to both sides of the equation to give Complete the square on the left hand side. Remember to take out a common factor of 2 first. \[\begin{aligned} 2 x^{2}+12 x+5 &=2\left[x^{2}+6 x+\frac{5}{2}\right] \\\\ &=2\left[(x+3)^{2}-\frac{13}{2}\right] \\\\ &=2(x+3)^{2}-13 \end{aligned} \] Rearrange and solve the resulting equation for x. \begin{aligned} 2(x+3)^{2}-13&=0 \\\\ 2(x+3)^{2}&=13 \\\\ (x+3)^{2}&=\frac{13}{2} \\\\ x+3&=\pm \sqrt{\frac{13}{2}} \\\\ x&=-3 \pm \sqrt{\frac{13}{2}} \end{aligned} We have two solutions to the equation: \[x=-3+\sqrt{\frac{13}{2}} \\\\ x=-3-\sqrt{\frac{13}{2}} \] Incorrect sign in the middle of the perfect square bracket The sign will always be the same sign as the coefficient of x. Errors with fraction arithmetic In particular, remember to square the numerator and denominator when squaring a fraction. Going the wrong way when working out the adjustment Remember to always work out how you get from the perfect square to the expression in the question. Forgetting the ± sign when using the completing the square method to solve quadratics Check carefully to make sure you haven't lost one of your solutions! Half the coefficient of x is 6 so it is (x+6)^{2} . Expanding (x+6)^{2}. gives us x^{2}+12x+36 so we do not need to add or subtract anything. Half the coefficient of x is 3 so it is (x+3)^{2} . Expanding (x+3)^{2} gives us x^{2}+6x+9 so we need to subtract 7 which gives us (x+3)^{2} -7 . =(x+\frac{5}{2})^{2}+\frac{23}{4} = (x-\frac{5}{2})^{2}+\frac{73}{4} =(x-\frac{5}{2})^{2}+\frac{23}{4} Half the coefficient of x is \frac{5}{2} so it is (x-\frac{5}{2})^{2} . Expanding (x-\frac{5}{2})^{2} gives us x^{2}-5x+\frac{25}{4} . We need +12 , which is the same as + \frac{48}{4} so we need to add \frac{23}{4} . Therefore the answer is (x-\frac{5}{2})^{2} + \frac{23}{4} . We must first take a factor of -2 out to get -2(x^{2}+2x-7) . We then need to complete the square for x^{2}+2x-7 . This gives us (x+1)^{2}-8 . So the final answer is -2[(x+1)^{2}-8]=-2(x+1)^{2}+16 . We need to start by completing the square. This gives us y=(x-3)^{2}-8 . We can then see that the minimum point is when   (x-3) = 0 so x=3 . Substituting this in gives us y=0^{2}-8=-8. . Therefore the minimum point is (3, -8) . Completing the square gives us (x+4)^{2}-13 . Therefore we need to solve: \begin{aligned} (x+4)^{2}-13&=0\\\\ (x+4)^{2}&=13\\\\ x+4&=\pm \sqrt{13}\\\\ x&= -4 \pm \sqrt{13} \end{aligned} 1. x^{2}+12x+19   can be written in the form (x+a)^{2}+b   where a   and b   are integers. Work out the values of a   and b . (2 marks) 2. (a)   x^{2}-6x+7 can be written in the form (x+a)^{2}+b   where a   and b   are integers. Work out the values of a   and b . (b)   Hence, or otherwise, solve the equation x^{2}-6x+7=0 Give your answers in surd form. (4 marks) (a) a = −3 (1) b = −2 (1) (b) x = 3 + \sqrt{2} (1) x = 3 − \sqrt{2} (1) 3. (a) 3x^{2}+12x-18   can be written in the form a(x+b)^{2}+c   where a   and b   are integers. Work out the values of a, b   and c . (b)   Hence, or otherwise, find the coordinates of the turning point of the graph of y=3x^{2}+12x-18 (4 marks) (a) a = 3 (1) b = 2 (1) c = −30 (1) (b) (−2, −30) (1) You have now learned how to: Complete the square for a quadratic expressionSolve quadratic equations by completing the squareIdentify turning points by completing the square Factorising Simultaneous equations Rearranging equations Prepare your KS4 students for maths GCSEs success with Third Space Learning. Weekly online one to one GCSE maths revision lessons delivered by expert maths tutors. Find out more about our GCSE maths tuition programme. We use essential and non-essential cookies to improve the experience on our website. Please read our Cookies Policy for information on how we use cookies and how to manage or change your cookie settings.AcceptPrivacy & Cookies Policy Completing the square is one of the most basic things you'll learn on an intermediate level in mathematics. This is a technique that we can use to transform a quadratic equation so that the left side is a perfect square trinomial. We can want to apply this technique for many different reasons. In many cases it will be to help us simplify the equation, so that we can work through the math easier. So, whenever you are asked to find the roots, it means that you have been asked to find the value x where the value of y = 0. Y = 4x2 + 40x + 280, 0 = 4x2 + 40x + 280 The first step needs to be about making the equation simpler so that it is easier to solve. If we divide both sides by 4, we get: x2 + 10x + 70 Now time to make a perfect square. Here you can create assumptions. We can add and subtract terms as well. For example: x2 + 10x + 25 - 25 + 70, 0 = (x + 5)2 + 45 Subtracting 45 on both sides, - 45 = (x + 5)2 + 45 - 45, √- 45 = (x + 5)2v There are no roots possible for the above equation as the root is in minus and therefore means no real number will exists for the value of x. As we have discussed on our previous topic page there are several different methods you can use to solve quadratics. When do you use the method that we are exploring here? The overall answer is that it depends on what you are trying to grasp about the quadratic. In most cases we use this method to determine the roots of the equation. This is because this method does not have a need to use rational factors. If you are looking for complex roots, this is your go to technique. If the quadratic present itself in proper form or you can easily manipulate it to be in proper form (a2 + 2ab + b2), then it is already square. This method can be used rearrange an equation to make it easier to work with. As you move on to advance quadratics, this method will simply be used to help you rearrange equations to make the math easier to work with. Who doesn't want to work on something that is less complex? Page 2 Math Worksheets For All Ages Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. Solving quadratics via completing the square can be tricky, first we need to write the quadratic in the form (x+\textcolor{red}{d})^2 + \textcolor{blue}{e} then we can solve it. Since a=1, this can be done in 4 easy steps. Example: By completing the square, solve the following quadratic x^2+6x +3=1 Step 1: Rearrange the equation so it is =0 \begin{aligned}(-1)\,\,\,\,\,\,\,\,\,x^2+6x+3 &=1 \\ x^2 +6x +2&=0\end{aligned} Step 2: Half the coefficient of x, so in this case \textcolor{red}{d}=6\div 2=\textcolor{red}{3}, and add it in the place of \textcolor{red}{d} (x+\textcolor{red}{3})^2 + \textcolor{blue}{e} Step 3: Next we need to find \textcolor{blue}{e} which equals the constant at the end of the quadratic, +2, minus \textcolor{red}{d^2}, then replace \textcolor{blue}{e} in the equation (\textcolor{blue}{e} \textcolor{black}{=c-}\textcolor{red}{d}\textcolor{black}{^2} as \textcolor{black}{a=1}). \begin{aligned}\textcolor{blue}{e} &= 2 - \textcolor{maroon}{9} \\ \textcolor{blue}{e} &= -7\end{aligned} (x+\textcolor{red}{3})^2 \textcolor{blue}{-7} = 0 Step 4: Now we have the equation in this form we can solve the equation. \begin{aligned}(+7)\,\,\,\,\,\,\,\,\,(x+3)^2 -7 &= 0 \\ (\sqrt{})\,\,\,\,\,\,\,\,\,\,\,\,\,\,\,\,\,\,(x+3)^2 &= 7 \\ (-3)\, x+3 &= \pm \sqrt{7} \\ x &= \pm \sqrt{7}- 3\end{aligned} This gives the solutions to be \sqrt{7} - 3 and -\sqrt{7} - 3 Remember: A square root can have both a positive and negative solution Level 6-7GCSEAQAEdexcelOCRCambridge iGCSEEdexcel iGCSE Switch to our new teaching resources now - designed by teachers and leading subject experts, and tested in classrooms.These resources were created for remote use during the pandemic and are not designed for classroom teaching.View new resourcesLesson detailsThere are other methods to find the solutions of a quadratic equation.One of these methods is called completing the square.By representing this with a model, you can see why it has this name.Completing the square is useful when the quadratic cannot be easily factorised.The square root of a number can be positive or negative.Completing the square - Completing the square is the process of rearranging an expression of the form ax^2 + bx + c into an equivalent expression of the form a(x + p)^2 + qThat we subtract the constant squared when the constant in the bracket is positive and add the constant squared when the constant in the bracket is negative.Squaring a negative value gives a positive value. Whether the constant in the bracket is positive or negative, the square of the constant must always be subtracted. This can be made clearer with algebra tiles or area models.In the lesson, the algebra tiles are orientated on a pair of axes so that negative tiles are in the second and fourth quadrant. This is not essential for understanding but does help pupils see where partial products are positive and negative and makes the algebra tiles easier to place correctly.Which of these is a solution for $$(x - 3)^2 = 0$$? Correct answer: $$x = 3$$Which of these is a solution for $$x^2 = 0$$?Correct answer: $$x = 0$$Which of these is a solution for $$(x - 4)(x + 2) = 0$$?Correct answer: $$x = -2$$Correct answer: $$x = 4$$The equation for this curve is $$y=x^2 + 3x + 3$$. How many real solutions will it have? Rearrange the following equation to make $$m$$ the subject: $$3x - m = 9$$Correct answer: $$m = 3x - 9$$Rearrange the following equation to make $$x$$ the subject: $$3x - m = 9$$Correct answer: $$x = \frac{m + 9}{3}$$Write $$x^2 + 4x + 5$$ in the form of $$(x + p)^2 + q$$Correct answer: $$(x + 2)^2 + 1$$Express $$x^2 + 6x + 10$$ as $$(x + p)^2 + q$$Correct answer: $$(x + 3)^2 + 1$$Express $$x^2 + 8x + 14$$ as $$(x + p)^2 + q$$Correct answer: $$(x + 4)^2 - 2$$Put $$x^2 - 4x + 5$$ in the form $$(x + p)^2 + q$$Correct answer: $$(x - 2)^2 + 1$$Solve by completing the square: $$x^2 + 6x + 8 = 0$$Correct answer: $$x = -4, x = -2$$Solve by completing the square: $$x^2 + 6x + 5 = 0$$Correct answer: $$x = -5, x = -1$$