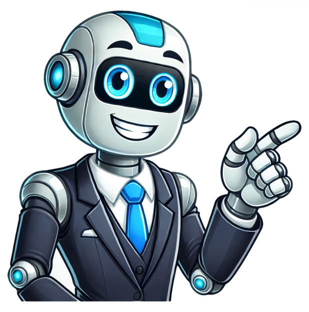

 A tessellation is a pattern created with identical shapes which fit together with no gaps or overlaps.Examples of tessellationsClockwise from top left: a pineapple, a turtle, Giant's Causeway, a honeycomb.Tessellations in nature have inspired tessellations in building, construction.Building and construction inspired by tessellations.Since the sum of the angles in a triangle is 180°, three identical triangles can be placed along a straight line.Three identical triangles in a row.By copying and rotating this pattern, six identical triangles can be placed together at a point leaving no gaps.This pattern can be repeated leaving no gaps.This is a tessellating pattern.All triangles tessellate.Prove that this triangle tessellates?Three identical triangles will fit together on a straight line since:$37^\circ + 68^\circ + 75^\circ = 180^\circ$Six identical triangles will fit together at a point since:$37^\circ + 68^\circ + 75^\circ + 37^\circ + 68^\circ + 75^\circ = 360^\circ$Therefore, the triangle tessellates.Regular polygons will tessellate if the size of the angle is a factor of 360°.Equilateral triangles have angles of 60°.$360^\circ \div 60^\circ = 6$6 equilateral triangles will fit together at a point with no gaps or overlaps.Equilateral triangles tessellate.Squares have angles of 90°.$360^\circ \div 90^\circ = 4$4 squares will fit together at a point with no gaps or overlaps.Squares tessellate.Do regular pentagons tessellate?Regular pentagons have angles 108°.$360^\circ \div 108^\circ = 3.333333...$3 pentagons leave a gap.4 will overlap.Regular pentagons do not tessellate.Do regular hexagons tessellate?$360^\circ \div 120^\circ = 3$3 regular hexagons will fit together at a point with no gaps or overlaps.Regular hexagons tessellate.Polygons with angles bigger than 120° will NOT tessellate.Language:EnglishCymraegGaeilgeGàidhlig Have you ever studied a pattern and been mesmerized by it? Were you taken away by its beauty or did you just get lost in it? This week we're exploring patterns in the form of a tessellation, which is a tiled pattern of one or more geometric shapes without overlapping or gaps. Tessellating Shapes What does it mean to tessellate a shape? Tessellations are patterns of shapes that fit neatly with one another. These patterns can consist of simple and complex shapes and can be two-dimensional, three-dimensional, or four-dimensional. Curious to see a few more examples of tessellations? Look at this children's math picture book with tessellations by Emily Grosvenor. It includes coloring pages, DIY instructions, and other resources. Learn how certain regular polygons, or many-sided shapes, can be tessellated with a few clicks. If you'd like to use paper and pencil, try making a tessellating bird pattern or tessellating stamps. If you're feeling particularly crafty, consider making origami flowering tessellation. Now that you're in the know about tessellations, celebrate this intersection of math and art annually on Tessellation Day, June 17. Mark your calendar to celebrate it in 2021 with a unique tessellation of your own. Tessellations in Nature Tessellations can be made and they can also be found in nature. Repeating patterns can be found from the ground, such as snake skin, to the sky, with dragonfly wings. Snakes have a repeating pattern within their scale patterns, while dragonflies have a repeating pattern within their wings. Their tessellations are different, but both share the purpose of helping their species survive. Tessellation also has many different purposes within the natural world. Tessellation allows spiders to create webs to eat, while honeycombs allow bees to survive. What are some examples of tessellation in your backyard? Or your local park? Noteworthy Tessellations M.C. Escher, known as the father of tessellations, may not have been the first to tessellate, but is certainly famous for many of his graphic images consisting of patterned geometric figures. He once said, "We adore chaos because we love to produce order." Take a look at some of Escher's well-known art. Itching to check out more tessellating shapes? Here's a compilation of tessellations from artists around the globe. Read up on tessellations in history, mathematics, and four dimensions. Looking to level up on your tessellation game? Learn about Heesch numbers and tiling. Artist of the Week Toyin Ojih Odutola is a Nigerian-American visual artist. She graduated from the California College of San Francisco and she had her first solo exhibition in 2011. Her exhibition focused on black ballpoint ink on white backgrounds. She communicates through texture, her various marks representing dialect and accent. Odutola was inducted into the National Academicians Class of 2019, which honors artists who have made remarkable contributions to American art. Share We'd love to see the results of your experiments! Tag @skokielibrary when you share photos of what you've created on social media. Written by Erica and Veena. Tessellations are a fun, hands-on way to explore STEAM, whether you are in art class, math class, or in a STEM or STEAM classroom. Certain basic shapes can be easily tessellated: squares hexagons triangles Combination shapes, complicated shapes, and animals such as the ones found on these sites are also examples to print and color: Shapes that Tessellate Lizards, M.C. Escher and more What exactly is a tessellation? Tessellations are patterns resulting from arranging, or tiling, shapes without any gaps. They can be made by positioning the same shape with one of these three operations: translation rotation reflection Translation can be thought of as sliding the shape along a plane. This is the type of tessellation you can make easily with a sticky note (as shown below). Rotation tessellations are accomplished by (you guessed it!) rotating the tessellated shape. Reflection tessellations are mirrored. You can also create complex tessellations by combining multiple operations. Create a Translation Tessellation Materials needed: square piece of paper (a small sticky note works well) scissors tape paper pencil There are some videos for making rotational and mirror tessellations on YouTube once your students have mastered the simpler translation tessellation: Rotational tessellations Reflection or Mirror Tessellation Use a Collaborative Tessellation for a Research Project I had so much fun creating artistic tessellations with my kids that I created a simple "I" tessellation research project for inventions! A list of 50+ inventions is included that students can research and report on in a fun way. You can find the invention tessellation resource here. Inventions Tessellation Project Free Download – Print and Color Tessellation Pages If you want to relax and color, print these at up to 4 pages per sheet (there are 5 pages included): Pin these ideas to save them for later: It's incredible how a tessellation design transforms a blank piece of paper into an exciting and beautiful design. I was amazed when I saw for the first time how a simple design transforms into a mesmerizing and stunning pattern. Creating tessellations is for all ages and more straightforward than it looks. Art and math teachers use tessellations in their lessons, art projects, and inspiring STEAM-related exercises. There is even a World Tessellation Day on June 17. Would you like to make your own tessellation art? This guide covers easy step-by-step instructions for all ages to do tessellation craft projects. It's not an art lesson, but the introduction to the no-cut and cut-method are ways to create projects for kids to make their tessellation artwork at school or home. Tessellation forms a pattern by repeatedly arranging tiles or shapes without gaps or overlays between the shapes – think mosaic pattern tiles and how jigsaw puzzles interlock. A tile refers to the repeating tessellation shape. Simple tessellation patterns have a basic design using a geometric shape like a square or triangle; they can also be more complex using irregular shapes. A chessboard is an example of a simple tessellation; the squares meet side to side without gaps. The geometric-shaped tile must tessellate itself and fit precisely in the tessellation. Tessera in Latin means a small stone cube, square tablet, or die; in Ionic, Greek tesseres means four. The first tilings were made from square tiles, and the Romans used mosaic pieces to create artistic tiles and floors. Tessellation has various meanings: A picture made from small square tiles is the original meaning An image made from tiles of different shapes, not just squares Tile-sized uniformly shaped pictures Using non-square tiles to fill a space without gaps or overlaps A picture created from repeating a single tile without gaps or overlays in a 2D or 3D space (M.C. Escher-style). A regular polygon like a square, equilateral triangle, and a hexagon can tessellate. Rectangles, rhombus, and trapezoids also tessellate. Isosceles triangles tessellate on two sides, and an octagon has limited tessellation options. A circle and a pentagon cannot tessellate; these shapes leave gaps. The types of polygons classify tessellations as regular, semi-regular, and irregular or non-regular. History Of Tessellation Art The Sumerian culture, about 5000 years ago, used tessellations to decorate columns. Almost every civilization created tessellations in one form or another—cultures like the Chinese, Indian, Arabic, and Irish practiced tiling in various complexity. The German mathematician, Johannes Kepler's documentation in 1619 are some of the earliest studies on tessellations. He was the first to explain the hexagonal structures of snowflakes and honeycombs; he also wrote about regular tessellation and semi-regular tessellation. The Russian scientist Yevgraf Fyodorov was the first to study tessellations mathematically. Studying the arrangement of atoms in crystalline solids, Fyodorov proved that every periodic tiling features one of seventeen different groups of isometries. The 20th-century artist Maurits Cornelis Escher, born in 1898, is one of the most famous graphic artists in the world. He was known for detailed realistic prints that achieved unusual optical illusions and conceptual effects. Studying at the School for Architecture and Decorative Arts in Haarlem, Netherlands, MC Escher became interested in graphics. Although he struggled with maths at school, he made mathematically inspired woodcuts, mezzotints, and lithographs. Inspired by nature, Escher studied insects, plants, and landscapes using the details in his artworks. During his travels in Spain and Italy, he was fascinated by the mathematical structure of architecture, Moorish Alhambra Palace's tilings, and the Mezquita of Cordoba. M.C Escher became well-known among mathematicians and scientists, but not as much as an artist until 70 years old; a retrospective exhibition made him famous in the art world. In the 20th century, he became widely know, and in the 21st century, he was celebrated with global exhibitions. The National Gallery of Art collection includes more than 400 works by Escher; it has the preeminent collection outside Holland. His most famous images are Drawing Hands, where each hand draws the other, and Hand with Reflecting Sphere, a self-portrait holding a crystal ball. Unlike M.C Escher, who designed on paper, Rober Fathauer designed directly on a Macintosh computer. Born in Illinois in 1960, Fathauer was interested in art from an early age. He received the Bachelor of Science with a double major in Mathematics and Physics from the University of Denver and his Ph.D. in Electrical Engineering from Cornell University. Dr. Fathauer was a follower of MC Escher, the most famous tessellation artist in history, and was inspired to create a variety of tessellations. In 1993, he founded a business, Tessellations, producing tessellation puzzles, mathematics manipulatives, books, and classroom posters. He started creating tessellations on the computer for better expression of his intricate designs. His art, which inspired mathematicians, included tessellations, fractals, illusion, symmetry, and knots. M. C. Escher popularized nested shape tessellations. Since then, many artists worldwide create beautiful tesselations art, for example: A regular shape in mathematics means a shape with equal sides and equal angles. Squares, hexagons, and equilateral triangles are regular shapes that create regular tessellations. A semi-regular tessellation (also called Archimedean) consists of two or more regular polygons of the same length. With the same arrangement of polygons at every vertex (corner where they meet), there are eight types of tessellations. They comprise different combinations of equilateral triangles, squares, hexagons, octagons, and dodecagons. Each tessellation is named for the number of sides surrounding each vertex. These tessellations are named according to the number of sides of the shapes surrounding the vertex. Where multiple regular polygons form a semi-regular tessellation, an irregular tessellation isn't made from regular polygons. These irregular figures can create an infinite number of irregular tesselations. Monohedral tiling use only one shape that rotates or flips to form the patterns. Mathematicians call such a shape congruent. Three-sided and four-sided shapes tesselate in at least one direction. Form a dual of a regular tessellation by taking each polygon's center as the vertex and joining the centers of adjacent polygons. A square tessellation is its own dual; hexagonal and triangular tessellations are duals of each other. Create the dual by drawing a dot in the center of the polygon. Connect all the dots and erase the original pattern. Aperiodic tessellation designs are non-repeating patterns using shapes like pentagons or spirals. The English mathematical physicist Robert Penrose, who shared the 2020 Nobel Prize in Physics with Reinhard Genzel and Andrea Ghez, is also famous for his Penrose Tiling. He created aperiodic tilings with pentagons that could cover an infinite area without overlapping, leaving a gap, or repeating the pattern. He used two types of symmetrical tiles to create infinitely changing patterns – the most famous are the kite-and-dart shapes. The symmetry of geometric patterns is seen in the repetitive repeats of the original shape repeated in the completed tessellation. Escher described four ways to move a motif to another position in the pattern creating different types of symmetry. This type is also known as a translation tessellation. The shape isn't flipped or rotated with translational symmetry, but the cut-out shape slides or translates across the paper to the opposite edge. The shape stays in alignment with the original shape to make a successful translation tessellation. A reflection tessellation flips the shape to the left, right, top, or bottom; it can also flip at an angle. If you draw a line down the middle of this "simple" tessellation, the two parts are symmetrical images, mirroring the other. A rotation tesselation turns or spins the shape around in a circular way around the fixed central point. Rotations have a center and an angle of rotation. Glide reflection tessellation uses reflection and translation concurrently. There is no rotational symmetry or reflectional symmetry. Middle school-aged students can create striking tessellations at home, an art lesson at school, or a math lesson using the tessellation technique. This introduction to tessellations describes three ways to create unique tessellations that are great projects for kids. Piece of Paper For no cutting method – one-half of 8 x 5 inches For 1-cut process – 5 equally sized square sheets of paper For 2-cut method – 4 x4 inch square piece of paper and drawing paper Pencil Scissors Tape Markers or Crayons in a variety of colors These beginner tessellation activities are easy; they don't even need a tessellation tutorial; no cutting is necessary. That makes this one of the best projects for kids- even young ones. Start with half of a regular 8.5 x 11-inches white sheet of paper. Use the pencil to create equal width rows, about 3-4 rows. Draw with the pencil a basic shape in the first row. Make sure it covers the height of the row. Use shapes like a square, parallelogram, or triangle. Repeat the shape next to the first one making sure the shapes fit together with no gaps or overlapping. Draw a third identical shape next to the second shape. Do they still fit without gaps? If yes, continue drawing the shapes until the entire sheet is full. Color in your tessellation. Display your finished tessellation as a wall decoration on the fridge, or decorate a room with your design. Create a more complex tessellation using shapes like arrows, fish, flowers, 6-point star shapes animal tessellation. Beginners and children starting to use scissors will quickly master the 1-step cutting method; it is slightly more challenging than the no-cutting method. Teachers can assign exciting projects for kids in middle school using this technique. Start with five equally sized squares of paper. Take one square and cut any shape out of it by starting and ending on the same side. Line your cut-out on top of the second square of paper. Make sure it is aligned along the edges. Trace your cut-out onto the second square with a pencil. Make sure the cut-out faces the same direction. Repeat steps 3 and 4 for each square of paper. Cut out the tracing on one sheet of paper. Tape the flat edges of the cut-out shape and square paper (the straight edge) together. Make sure the adjacent edges and corners of the adjacent tiles line up. Repeat steps 6 and 7 for all the squares making sure the cut-out tracing's orientation is the same as the others before taping it. Fit all the pieces together like a puzzle forming one picture. Color the pieces to make your tessellation your unique work of art. This 2D tessellation technique is for the more adventurous students or kids looking for a challenge. The method adds an extra step to the 1-cut technique. Use a 4 x 4 inch of paper to make your tessellation shape. Start by cutting a weird shape out of one side of the square. Tape the cut-out shape to the opposite side of the square. Keep the orientation and make sure the square and cut-out's flat edges line up. Cut another weird shape out of one of the other two straight edges of the square. Tape the second weird shape to the opposite side of the square in the same way you did with the first cut-out. Trace your tessellation shape on the drawing paper repeatedly by fitting the shapes together without gaps or overlaps. Color, paint, or sparkle your design, making it unique and beautiful. You can copy and compile various finished tessellations to create coloring pages for more coloring fun. For inspiration, here are some cool ideas: For more fun activities, here are some great tessellation toys for kids: Tessellations are everywhere, in nature, and made by humans. Some great projects for kids in your classroom begin by having students identify tessellation in the world around them. Here are some make your own tessellation examples to get you started: Manufactured: brick wall, kitchen floor with square tiles, glass roofs, hexagon paving, fencing, tweed material Nature: honeycomb, snake's skin, leopard skin, turtle shells, fish scales, dragonfly wings, mud cracks Sports: soccer balls, athletic shoe treads, basketball nets Food: raspberries, pineapples, orange segments, corn Plants: plant cells in wood, plant cell structures, sunflowers Tessellation patterns occur in art, architecture, and various types of tiling puzzles ranging from traditional jigsaw puzzles to modern puzzles based on mathematics. The tessellation or tiling must tile a plane infinitely without gaps or overlapping. The tiles must be regular polygons, shapes with interior angles that add up to 360 degrees. You can create regular tessellations from six-sided or fewer polygons. Each vertex where the corners meet must look the same. The study of tessellations shows the integral part it plays in mathematics. No matter your math skills, all students can make tessellations; it's a fun way to inspire math activities. Here are a few examples of tessellations used for math exercises: Measure the distance by establishing the length of the sides of the tiles, individual tiles, or the entire design. Calculate the perimeter of the tessellation tiles or shapes. Discover different possible shapes they can make. Identify shapes with specified sizes. Explore how many different shapes they can make within a specified area. Calculate the area of a large design. Find the symmetry lines in the tessellation design, the design with most lines, or designs with a specified number of lines. Young kids learn about tessellations and shapes like squares, triangles, rhombus, hexagon, and rectangular blocks. Middle school kids can learn pentagonal patterns by tessellating them, discover the sum of angles at any one vertex is 360 degrees. High school pupils' activities could include discovering and proving which shapes tessellate and that the angle measure at any point must be 360 degrees. Where do you see tessellations in everyday life? Have you made a tessellation? What is your favorite shape? With all my thanks to David Bailey tess-elation.co.uk for its invaluable assistance concerning the translation. Summary 2 Transforming the polygons 7 Tessellating with words 8 Distorting the structures 9 Playing with isometries Foreword "The inventor has, all of a sudden, the distinct feelingthat the designs which he creates (...)already existed before to have never been thought in the human brain. Louis de BROGLIE My song is really ended when she looks to be made itself alone. Georges BRASSENS I have such sensations all the time when I am working on designs for regular surface fillings. It seems it is not I who am doing the creating, but rather that the innocent flat patches over which I am slaving have their own will, and it is they which guide the movement of my hand as I draw. Maurits Cornelis ESCHER "A long time ago, I chanced upon this domain in one of my wanderings I saw a high wall and a premonition of an enigma, something that might be hidden behind the wall, I climbed over it with some difficulty. However, on the other side I landed in a wilderness and had to cut my way through with a great effort until - by a circuitous route - I came to the open gate, the open gate of mathematics." ..."I walk around all alone in this beautiful garden, which certainly does not belong only to me, but whose gate is open to everyone." Maurits Cornelis ESCHERPeriodic filling of a plan This method, taken from my book "PARCELLES D'INFINI", offers a walk in the garden of the regular division of the plane that Escher explored in many directions. This area may seem limited judging by the few artworks coming from others. I hope that what follows will make you realize that there are an infinite number of possible figurative motifs. And it is not just copying Escher to make a periodic division of the plane. ... No more, for instance, than copying Jean-Pélerin* that makes a drawing in perspective. And if, as I hope, you are inspired to do figurative 'tiles', throughout the following pages you will find how to give birth to them. And when, after some difficulties, you successfully make one, you will experience a great feeling of joy and humility before this "PARCEL OF INFINITY". *. Called the Viator. He wrote the first printed treatise on perspective, published in 1505. Return to summary There is so much art in naturethat very art consists to well heard and imitate it. Jacques Bénigne BOSSUET It is a sad thing to think that nature speaksand that the human race is not listening. Victor HUGO he laws of men are ephemeral, those of nature are eternal. Even if our incursions on its paths will be always modest, this remains a great pleasure and a great honour that to be able to imagine plane variations on its own structures. The honeycombs of the bees, the fish scales or the sunflowers give us some fine examples of tessellations but it is especially the crystals that nature offers us to show us the way. We find in their structure the 17 symmetry groups allowing to repeat a pattern at infinity. But to draw figurative tessellations it would be preferable to create to us a both simpler and more appropriate classification. This is what we will do. There are three regular polygons that can divide the plane periodically, these are: The triangle The square The hexagon But fortunately, quantities of irregular polygons can also divide the plane periodically. A few examples: The triangle The right isosceles triangle The rhomb 2 angles120° The rhomb The parallelogram The quadrilateral The pentagon 2 sides parallel and equal The hexagon 2 opposites sidesparallel and equal, between 2 x 2 sides adjacent and equal In addition, it is possible to replace the sides of all polygons dividing periodically the plane, by compensated deformations that does not alter their area. Examples : Translations of compensated deformations on parallelogram Translations of compensated deformations on hexagon with opposites sides parallel The multiplicity of achievable compensated deformations brings the amazing opportunity to give rise to an infinite number of figurative motifs. .Examples : Progressive translations of compensated deformations on squares Transformations by translations of a same rectangle into six different birds The base polygon having undergone compensated deformations is named tile. Examples : • Translations on a parallelogram: Base polygon Compensated deformation Tile • Translation on a square: Base polygon Compensated deformation Tile • Translation on a concave hexagon: Base polygon Compensated deformation Tile • Translation on a regular hexagon: Base polygon Compensated deformation Tile Glow Foxes at the Moon Brotherhood Leaping witch doctor Karate A squared paper facilitates the tracing of the tile. Example : Base polygon (this is for us a hexagon of which 2 x 2 sides are in prolongation)Compensated deformations Tile A set of tiles is named tessellation : Lucky Blanchepatte, my dear gone dog 2 TRANSFORMING THE POLYGONS Return to summary The geometry is to fine artswhat grammar is to the art of the writer. Guillaume APOLLINAIRE More the art is controlled, limited, workedand more it is free. Igor STRAVINSKY rying to give birth to figurative motifs without geometric bases gives very limited results. The polygons are the eggs from which will hatch a multitude of characters. There are three possible deformations that can replace the sides of a polygon: 1. Any (as its name says): side of polygon before deformationside of polygon after deformationsymmetry axiscenter of rotation 2. Axial (any deformation more its reflection in relation to an axis): 3. Rotatory (any deformation more its 180° rotation): The deformation, of any or axial of a side must always be compensated by an identical deformation on another side. The rotatory deformation compensates itself. We call these compensations isometries. There are four types of isometries : 1. The translation is the simple rectilinear slide of one deformation: 2. The rotation rotation is either rotatory deformation or the pivoting of a deformation around a centre of rotation: 3. The glide reflection is the reflection of a deformation in relation to an axis, followed by a translation: 4. The symmetrie is the reflection of a deformation in relation to an axis: Examples : • Two translations on a parallelogram: • A 180° rotation and another to 60° on an equilateral triangle: • A glide reflection as well as a translation on a parallelogram: • Two symmetrical slide reflections and an axial translation on hexagon: The glide reflection requires some clarification: - The sides of the polygon may be adjacent (have a common end) or non-adjacent : A polygon can have two glide reflections. In this case, their vectors axes are either parallel or perpendicular : The tracing of theglide reflections is particularlysimplified ona sheet of squared paper Let L be the length of the sides of the polygon: The v and v' points allow to trace the vector axis of the glide reflections. This vector axis indicates the axis of reflection as well as the direction and length of glide. Example : The base polygon of this bird is a concave pentagon having one rotatory side and 2 x 2 sides of perpendicular vectors axes. ISOMETRIES DESIGNATIONS AND ABBREVIATIONSCONDITIONS IN THE POLYGONSIMPLIFIED REPRESENTATIONS The translationT2 sides parallel and equal The rotation - 180°R21 any side - 120°R32 sides equal and adjacent at 120° - 90°R42 sides equal and adjacent at 90° - 60°R62 sides equal and adjacent at 60° The glide reflection(If there are 2 in the polygon, we add ' to the 2nd) G2 equal sides The symmetryS2 symmetrical sides There are four types of polygons that can divide the plane periodically: the trianglethe quadrilateralthe pentagonthe hexagon The systematic search for all possible combinations between the four isometries and the four types of polygons allows us to lead to a large amount of specific polygons of which we can eliminate those unable to fill the plane. Examples : There then remains 81 specific polygons, called isohedral tiles, of which we also eliminate those where the possibilities of obtaining figurative motif are too small: - because having one or more rectilinear sides: detained because removed because - because having central symmetry: Ultimately, there remains 35 modal tiles; 35 parcels of infinity that the magician in you which lies dormant maybe, and will transform into a rabbit, butterfly or dove. Doves The need for a framework to this method should not let us forget that there are always opportunities to get out of the framework. Examples : • Escher's splendid "Chinese" has two rectilinear sides: • Half card-trick has a central axis of rotation: Half Card-trick Here is its basic polygon: • Same for the octopus: Return to summary It happens sometimes that a person (...) feels born and grow in her, one day, a desire very conscious to approach by the imagination, infinity as close as possible in the purest way.Maurits Cornelis ESCHER Hope nothing from human if he works for his own life and not for eternity. Antoine de SAINT-EXUPÉRY The feeling of infinity is the real human attribute. Madame de STAËL ssembling the tiles, it is to arrange parcels of infinity on eternal structures. Assembling the tiles, it is the privilege to play jigsaw puzzles with the universe. Tiles similar to a modal tile can fit after rotation and/or glide reflection. They then have a different orientation. Examples : Glide reflectionRotationGlide reflection + rotation This set of tiles can then similarly fit in turn by translation, and can then be continued to infinity : All these similar tiles are the transformations of the base modal tile. The whole, we have seen, is called tessellation. This round of lovers is just waiting to grow! Visually, what characterises first a tessellation is the number of directions taken by the motifs. There are six possible directions, and therefore will be six original types. Type 1 will have all its tiles in the same direction. Type 2 will have its tiles head to tail. Type 3 will have its tiles in three directions, type 4 in four directions and type 6 in six directions.Examples : • Type 1, the athlete: • Type 3, the lizard: • Type 4, the kitten: • Type 2, the siren: • Type 6, the otary: Then, the tessellations with one or two directions may have their motifs reflected. There will be also a type 1G (G for Glide reflexion) and a type 2G. • Type 1G, the rooster: • Type 2G, the dog: Finally, tessellation types 1, 2, 3 and 4 may have their motifs symmetrical. We will have in addition type 1S, type 2S, type 3S and type 4S. • Type 1S, the gorilla: • Type 2S, the butterfly: • Type 3S, the duck: • Type 4S, the frog: That is to say 11 types in totality, which will find themselves on 11 structures. In addition, these 11 types have subtypes classified according to their basic polygon. In total, we meet again the 35 modal tiles of the previous chapter. • The base polygon of the kitten is a right isosceles triangle. We will designate it by the isometries of the sides, that is to say R2 R4 R4: • The one of the frog is a square. Designation R4S R4S R4S R4S: Now here are the designations of basic polygons for: • the athlete: T T T T T T • the siren: R2 R2 R2 • the lizard : R3 R3 R3 R3 • the otary : R2 R6 R6 • the rooster : T G T G • the dog: R2 G G • the gorilla: T GS GS T GS GS • the butterfly: T R2S R2S T R2S R2S • The one of the duck is a rhomb with two angles of 120°. We will refer it by R3S R3S R3S R3S because the sides are both of 3 rotations and symmetries: The base polygon of the 'madman' is a regular hexagon. Designation R3 R3 R3 R3 R3 R3: The use of isometric paper is ideal for tracing tiles with isometries of rotations 3 or 6. Kittens Ducks Chessboard with frogs The madman's jigsaw puzzle • The base polygon of the "Li'l strong man" is a square. We will designate it by R4 R4 R4 R4: • The one of Escher (Oh yes! even the Escher's profile is a tile!) is a hexagon. Designation T R2 R2 T R2 R2: Return to summary If there is no trick, it is strong...If there is a trick, it is even stronger!MYR et MYROSKA All things, near or far,secretly are connected to each otherand you cannot touch a flower without disturbing a star. Francis Joseph THOMPSON he structures are similar to the scenes of magic theatres, in that they contain all the secrets needed for representation. To know these secrets disenchants only the fool. We have seen that there are 11 structures, but the crystallographers count 17 symmetry groups. In fact, the six missing groups do not allow us to create unique motifs. We will indicate all the same the 6 groups of structures that do not allow to make unique motifs, this after the first 11 groups The structures are represented using the following elements: - center of rotation : •- symmetry axis : ——— — ——— — ——— - axis of glide reflection : ——————— TYPE 2G (pgg) Rotation 180° + glide reflexion + translationStructure + base polygon and its transformations before translationsExample of base tile Example of figurative tileExample of arrowhead andcolored tessellation(minimum of colors) b (IH53) Quadrilateral R2 R2 G G36 / 412 / 77 / 98 / 116 The symmetry group used is that of the International Tables for x-ray crystallography.The isohedral type defined is the one as given by B. Grünbaum and G. C. Shephard in their book Tilings and Patterns.When we have several kinds of transformations before translation as in the present case, we try the first: this gives us two tiles. Then we make: - either the transformation of the two tile set (top example); - either the transformation of each of the two tiles separately (example below with concave tiles). The small numbers indicate the angle of rotation (2 = 180°, 3 = 120°, 4 = 90°, 6 = 60°).The structure of type 1 will be vacant because it contains only translations.Nineteen basic polygons may be concave. These are: 1b, 1Gb, 1Gc, 1Gd, 1Sb, 2b, 2d, 2e, 2Gb, 2Gc, 2Ge, 2Gf, 2Gg, 2Gh, 2Sb, 2Sc, 3b, 4c et 6d. Exemple : The figures indicate the number of adjacent vertices. The 35 types of tessellations will be represented by 35 birds. All isohedral tessellations can be coloured with a minimum of two or three colors. If all adjacent vertices are of even numbers, two colors are sufficient. If there are vertices of odd numbers, three colors are needed. TYPE 1 (p1) Translations Structure + base polygonbefore translationsExample of base tileExample of figurative tile Example of arrowhead and colored tessellation (minimum of colors) a (IH41) Parallelogram T T T T38 / 472 / 482 / 492 / 502 / 522 / 73 / 74 / 80 / 105 / 106 / 127 / 128 b (IH1) Hexagon T T T T T T182 / 222 / 272 / 282 / 292 / 302 / 722 / 822 / 842 / 872 / 922 / 1112 / 1122 / 1132 / 1142 / 1202 / 1212 / 1292 TYPE 1G (pg) Glide reflexion + translations Structure + base polygon and its transformation before translationsExample of base tileExample of figurative tile Example of arrowhead and colored tessellation (minimum of colors) a (IH43) Parallelogram T G T G31 / 32 / 7112 / 97 / 108 / 109 b (IH44) Kite * G G G' G'19 / 62 / 66 / 67 / 762 / 96 / 102 / A13 *. A kite is a quadrilateral whose one diagonal is perpendicular to each other in the middle). c (IH2) Hexagon T G G T G' G'242 / 262 / 78 / 1102 / 1262 / 1302 d (IH3) Hexagon T G G' T G' G17 / 342 / 612 / 632 TYPE 1S (cm) Translations Structure + base polygonbefore translationsExample of base tileExample of figurative tile Example of arrowhead and colored tessellation (minimum of colors) a (IH68) Rhomb GS GS G'S G'S *91 / A14 *. The sides with glide reflection being symmetrical, they are also and necessarily translations. b (IH12) Hexagone T GS GS T G'S G'S *A1 *. The sides with glide reflection being symmetrical, they are also and necessarily translations. TYPE 2 (p2) Rotation 180° + translations Structure + base polygon and its transformation before translationsExample of base tileExample of figurative tile Example of arrowhead and colored tessellation (minimum of colors) a (IH84) Triangle R2 R2 R251 / 95 b (IH46) Quadrilateral R2 R2 R2 R29 / 88 / 90 / 93 c (IH47) Parallelogram T R2 T R275 / 1152 d (IH23) Pentagon T R2 T R2 R2 Escher did not make tessellations of this type e (IH4) Hexagon T R2 R2 T R2 R21 / 5 / 6 / 7 / 8 / 11 / A12 TYPE 2G (pgg) Rotation 180° + réflexion glissée + translations Structure + base polygon and its transformations before translationsExample of base tile Example of figurative tileExample of arrowhead andcolored tessellation(minimum of colors) a (IH86) Isoceles triangle R2 G G Escher did not make tessellations of this type b (IH53) Quadrilateral R2 R2 G G36 / 412 / 77 / 98 / 116 c (IH51) Quadrilateral R2 G R2 G33 / 107 / 124 d (IH52) Rectangle G G' G G'39 / 682 e (IH25) Pentagon T R2 T G G f (IH27) Pentagon R2 G G' G G'16 / 462 / 592 / 602 g (IH5) Hexagon T R2 R2 T G G10 / 582 h (IH6) Hexagon R2 G R2 G' G G'2 TYPE 2S (pmg) Rotation 180° + translations Structure + base polygon and its transformation before translationsExample of base tileExample of figurative tile Example of arrowhead and colored tessellation (minimum of colors) a (IH66) Rectangle T R2S T R2S *40 / 117 *. The sides with rotation 2 being symmetrical, they are also and necessarily glide reflections. b (IH69) Kite R2S R2S R2S R2S *37 / 89 *. The sides with rotation 2 being symmetrical, they are also and necessarily glide reflections. c (IH13) Hexagon T R2S R2S T R2S R2S * Escher did not make tessellations of this type *. The sides with rotation 2 being symmetrical, they are also and necessarily glide reflections. TYPE 3 (p3) 2 rotations 120° + translations Structure + base polygon and its transformations before translationsExample of base tile Example of figurative tileExample of arrowhead andcolored tessellation(minimum of colors) a (IH33) Rhomb R3 R3 R3 R3 Escher did not make tessellations of this type b (IH7) Hexagon R3 R3 R3 R3 R3 R3 R321 / 25 / 432 TYPE 3S (p31m) 2 rotations 120° + translations Structure + base polygon and its transformations before translationsExample of base tile Example of figurative tileExample of arrowhead andcolored tessellation(minimum of colors) a (IH36) Rhomb R3S R3S R3S R3S *532 / 542 / 103 / 123 *. The sides with rotation 3 being symmetrical, they are also and necessarily glide reflections. TYPE 4 (p4) 3 rotations 90° + translations Structure + base polygon and its transformations before translationsExample of base tile Example of figurative tileExample of arrowhead andcolored tessellation(minimum of colors) a (IH79) Right isoceles triangle R2 R4 R435 / 118 / 119 b (IH55) Square R4 R4 R4 R415 / 23 / 104 c (IH28) Pentagon R2 R4 R4 R4 R414 / 20 / 423,5 TYPE 4S (p4g) 3 rotations 90° + translations Structure + base polygon and its transformations before translationsExample of base tile Example of figurative tileExample of arrowhead andcolored tessellation(minimum of colors) a (IH71) Square R4S R4S R4S R4S*13 / 452 / 86 / 122 / 125 *. The sides with rotation 4 being symmetrical, they are also and necessarily glide reflections. TYPE 6 (p6) Rotations 180°° + 2 rotations 120° + translat. Structure + base polygon and its transformations before translationsExample of base tile Example of figurative tileExample of arrowhead andcolored tessellation(minimum of colors) a (IH88) Equilateral triangle R2 R6 R644 / 94 / 99 / 100 b (IH39) Isoceles triangle R2 R3 R3 Escher did not make tessellations of this type c (IH31) Kite R3 R3 R6 R655 / 56 c (IH21) Pentagon R2 R3 R3 R6 R6572 / 70 / 79 DesignationStructureExample of figurative tileDesignationStructureExample of figurative tile pm(Minimum 2 motifs) p3m1(Minimum 3 motifs) cmm(Minimum 2 motifs) p4m(Minimum 3 motifs) pmm(Minimum 3 motifs) p6m(Minimum 3 motifs) Return to summary Imagination is more important than knowledge.Albert EINSTEIN The reason, it is the intelligence in exercise;the imagination, it is the intelligence in erection. Victor HUGO ou have the quill and the parchment. You have taken knowledge of the structures secrets. You now need to appeal to the most important: your imagination. If on an old decrepit wall you see figures appearing more or less of a fantasy nature, you have a good chance of success. When designing you can randomly choose a basic polygon and trace compensated deformations until you get a rough outline of motif. Or you may already have an idea of the motif to draw, trace it roughly, and adapt on it the type of best suited basic polygon and go back over the compensated deformations. Of course, you will need to use a lot of the eraser before succeeding with a figurative tile.Often your motif will be only half successful and so you will need to start over again.. But if really you get nothing satisfying, all is not lost. With the help of your computer, trace again your failed tile. Inside your tile add some motifs not connected, then select all and give a background gradient to all these traces, in the rainbow or tropical sunset style. It only remains to you to baptise the whole of a silly name and you just created a worthy representative of abstract art! Now that you know how to work the structures, the summary of the previous 35 basic polygons will help you to give birth to figurative tiles. Your hardware: squared paper, isometric paper for types 3 and 6, a pencil and an eraser. That's all! When you will want to multiply the tiles to make tessellations, a computer with a drawing program will be convenient. But that is not required, do not forget that Escher drew all his tilings by hand. Tessellation software like that of Kevin Lee (designer of TesselManiac) will also provide you with valuable assistance for the creation of tiles. Hexus(Abstract art!) type 1b Elbow to elbow Impossible icarian antipodisttype 2c Wrong way Samson's sleep(Modern art!) type 2c Beware... fir! Jealous After this little interlude of more or less successful tiles, we come back to our base polygons. • Let us take a square of type 1Ga. Trace a few broken lines on two adjacent sides (a). Then let us transfer them on the two other sides using a translation and a glide reflection (b). Let us study the resulting figure: It vaguely has a look of a little man. Let us reduce the left arm, this augments the right arm (c): Let us modify the left leg, and that improves the definition of the right arm and the right leg (d). Let us refine the exterior details taking account both of what would be the ideal line on one side and what would be the one of its isometry (e). Now let us draw the interior details of the little man. One realizes that we can still improve certain outside line elements (f). Now just realize of what we made: We have transformed a rigid square into a young boy who runs happily on our sheet of paper! Comings and Goings • Let us now take a rectangle, of type 1Ga, and draw a few lines. We soon get the silhouette of a marching service man: But his silhouette is leaning forward. Transforming the base rectangle into a parallelogram will enable us to straighten it out: And behold, from a rectangle at the beginning, we get a valiant service man who parades proudly below. • In fact, a good number of drawings of figurative tiles begin by being... some drawings! Let us take for example the goat below. Let us examine it. It appears clear that there is a possibility of a glide reflection between the front legs and horns: Then maybe another between a hind leg and the tail. It then only remains to adapt the glide reflections into a kite type 1 Gb: And here is a brave little goat that comes down from the mountain: After these three examples, perhaps it seems easy to you to draw figurative motifs. Don't believe it; you need to try and try again, hundred times on the drawing board start your work again. Respecful Salutes Mountain • Let us take again a square of type 1Ga and try a few lines: After the transformation of the square into a rectangle and then a parallelogram there appears an Indian, the hand raised as a sign of peace: • After the Indians, here are the cowboys with as base polygon a hexagon of type 1Gc: • Let us find again the historic figure of the rodeo cowboy in a concave hexagon type 1Gd: You will notice that the hat is detached from the rider and his horse. We have there, in fact, a tile composed of two subdivided tiles (see chapter Dividing the tiles). • Let us stay in America with the creator of rockabilly, Elvis himself, in this regular hexagon of type 1Gc: Indians Emulation Cowboy Rodeo • The rhomb type 1Sa is suitable to appear as the silhouette of a bear: • But if we lean the bear, the rhomb turns into a parallelogram of type 1a: • And if the bear starts to dance, the base polygon becomes a kite type 1 Gb: • The rectangle of type 2Gd easily turns itself into an undulating fish and could make a nice bathroom tiling: • From a sharp triangle of type 2 is born the pretty curves of a pretty girl: • The chess piece horse is developed from a quadrilateral of type 2b: Dive in Trouble Water Chess • This big good guy splendidly fills a hexagon of type 2Sc: • This parallelogram of type 2b turns itself into a quadrilateral to reveal a judo player at the heart of the action: But he has maybe a hidden face... Nice / Bad Judo • Also in action is this acrobat monkey born from a concave pentagon type 2Ge: • A type 2Gg hexagon is at the base polygon of the fish below. But only by its contours, as details of half of the fishes - which should be upside down - was drawn right way up. • This inflatable lizard is very convenient to be stored on the beach. Its underlying polygon is a hexagon of type 2Gh: • The green tree frog can come and go in a rectangle of type 2Sa: • Apart from its ventral fins, the angelfish fits perfectly to a kite type 2Sb: • This square of type 4b may give birth to a crested bird: Mimetisme Checkerboard with Birds • You can cut the jumping jack below and assemble it: Perhaps you notice that with the arms and legs in the air this one can also be born from a rhomb type 3Sa. So we can draw its transformations and thus constitute us with an army of jumping jacks ! Return to summary People never divert themself so well that at the pursuit of eternity.Jean GIONO If you want to progress towards infinity,explore the finite in all directions. Johann Wolfgang von GOETHE s cells divide themselves to multiply, why not make tiles to go two by two, see three by three or even more, to tessellate infinity? The tessellation with this hexagon will give the figure below. Three colors are needed: The tiles of all the types that we have seen can be divided into as many subdivided tiles as we want.Take for example a hexagon with opposite parallel sides of type 1 b: Let us divide the hexagon in several ways: In fact we can divide the base tile in as many different ways as we wish. Division causes changes in the vertices, and so in the minimum number of colors. We have seen that if all adjacent vertices of an isohedral paving are of even numbers, two colors are sufficient, and that if there are vertices in odd numbers, three colors are required. It is not the same for the subdivided tiles, as that may require up to four colors. Example type 1bExample type 3b • Here is a tile type 2Gc which resembles nothing much: But when it is divided into two it reveals a fish and a lizard: • During a walk, pick up a leaf. Draw its outline: Copy a third and a fourth still making them coinciding with another vertex. With tracing paper draw a second leaf by coinciding two of the vertices: With this method you can have the chance of seeing appearing a rabbit among the autumn leaves or a dove among the stars. The inaccessible star • Here are those of the Inaccessible Star: : • Here are the base polygon and the base tile of the 'rabbit among the autumn leaves': You may have noticed that the dove and the star above do not have quite the same outline that the dove at the bottom and the star of the top of the drawing does. It is this appearance/disappearance style of drawing dear to Escher that allows these approximations. These matters are sometimes even more pronounced in the drawings that follow. Dream of frog Prestidigitation Merry Christmas / Happy New Year Contrast Good and Evil Dream of Paper Hen Sea Wings and Sails Here is the base tile of 'Two Elements': Two Elements • This bird and this fish would be perfect in an appearance/disappearance style drawing, but the result would have been too close to Escher's Sky and Water. • Here is the base tile of Child's Eye View: : Child's Eye View This child look that might be the one of Albert Flocon – the brilliant engraver artist - which sets out the rules of the curvilinear perspective, this one giving a more logical and exciting image than the classic perspective for which we have structured space with straight lines that make it sad. "Learned since childhood, familiar for four centuries,the traditional image is credited finally for the real image.It is perhaps an imposture..."Albert FLOCON / André BARRELa Perspective curviligne And now, if we divide a few tiles into several lizards: • First of all here is the base tile of Lizards Squares, divided into three lizards including two that are identical: Lizards Squares • And here is the one of the opposite tessellation: • The Rose with Lizards is constructed by using tiles of type 4a and 6b between which we have made a few adaptations. Rose with Lizards • The drawing below has a type 3b base tile divided into four lizards: 7 TESSELLATING WITH WORDS Return to summary The words are chump change of thought.There are big talkers who pay us in dimes parts.Others, on the contrary, give only golden louis. Jules RENARD The art is beautiful when the hand, head and heart work together. John RUSKIN et us ask ourselves the question: could the motifs be other things than familiar figures, such as animals, objects or humans while still having recognizable contours? Well yes, of course! There are the characteres; these signs that men have imagined to communicate and which are, moreover, already in two dimensions.And even better: there are character sets that are the words and who have as well a specific meaning. We can choose a word for what or who it represents, for its value to us. We can draw it, work it, perfect it... love it as much as a poet. Why Escher did not make any tessellations with words remains a great mystery. The way has been indicated in a book by Scott Kim. Its title? Inversions. A unique book, at the border between calligraphy and mathematics. We find in, among other wonders, the word "figure" in white on a black background and which the space between letters gradually turns into black letters on a white background, again forming the word "figure" in negative. There is also the word "Lester", with the space between the letters forming the word "Pearl" (these are the Kim's parents). These are not tessellations but it is already good way along.It was tempting to go this way and make words in Kim's style. It was exciting to go at the end of the road and realize an old childhood dream: to make tessellations with words. And if possible with words dear to my heart.. We have seen that it is easy to divide a tile in two, three or more parts. Well, a tile-word is only the equivalent of a tile divided into as many subdivided tiles it contains letters, accents or dot on the i. It is the assembling of the base tile with its transformations and then the coloring that allows us to read the word in different colors. • There are no tessellations in the opposite example. But we could cut type 4b tiles like the one below: L'Infini (The Infinity in French) • Among several early attempts of tessellations with words, here is this one at right: An inconclusive attempt. But the result is interesting all the same. • Honor to the good master. Here is the type 4c base tile used for Escher Pavage (this means Escher Tessellation in French): Escher Tessellation The transformations of this base tile allow us to read the word "Escher" in four directions. • We find again this tile-word in decoration around Escher's portrait. Escher • Let us take again the tile-word "Escher". And let us color it as below. We see then to appear... Le Jardin d'Escher (The garden of Escher in French). Tiling example : The Garden of Escher • Here is the type 2e base tile of Kim : • Translation axes, rotation axes, symmetry axes, glide-reflection axes; the axes are present in all tessellations. Below you will find the type 6a base tile (equilateral triangle) of Axes: The transformations of this base tile allow us to read the word "Kim" in two directions. • En route to a ride to the Étoiles (Stars in French). Here is the used type 6a base tile: • Then here is the type 2c base tile of Inversion a particularly dear word to Scott Kim: Kim Inversion Axes Stars • Honor to the natural order, here is the type 4c base tile used for L'Ordre et le Chaos (The Order & the Chaos): • Yet on the contrary, here is the type 1b base tile of Paradoxe Jour / Nuit (Paradox Day / Night): • Of course, it must have a name of musician. To quote geniuses such as Bach or Mozart would certainly make a good impression... but it would not reflect my taste for good old Country music, music that I love and which brings me joy and satisfaction. And do not tell me that there are no geniuses in Country music. Merle Travis not only was one, but even more was a multi-talented genius. He excelled both as author, composer, cartoonist, watchmaker, historian, actor, writer, naturalist, impersonator, designer of guitars, singer and of course as a virtuoso guitarist. We are indebted to him for the guitar style that bears his name, the Travis-pickin'. A style in which the accompaniment, bass and melody are played at the same time. Here is the base tile of Travis: The Order and the Chaos Travis Travis Pickin' Paradox Day / Night • Brassens... Ah, Brassens! The man with a hundred masterpieces. The perfectionist of the "beautiful language" and chosen expression. The wonderful artisan engraver of words. That the word "Brassens" extends to infinity. • Depending on that it is white or black, the magic indicates either the art of producing some wonderful effects that are due to natural causes, or the sordid mystification by which some people claim to produce supernatural effects by the intervention of the spirits. Here is the type 1a base tile used in Magie blanche / Magie noire (White Magic / Black Magic): • Among the artists of white magic, Gérard Majax is a worthy representative of them, which does not hesitate to work against the phonies of illusion, the supposedly owners of extraordinary powers and others twisters of teaspoons. Here is his type 1b base tile: Brassens The Tall Oak White Magic / Black Magic Majax • Of course, here is the base tile of the unavoidable Infini (Infinity): • Then here is the type 2e base tile of Pavé (Tile!). First in two tones then into only one gradient tone. • This type 6a almost triangular tile is the one of Victorian tessellation: • If there is a name deserving to tessellate the infinite, it is that of "Einstein". Here is the used type 2e base tile: "The joy to contemplate and understand,this is the language that nature inclines to me."Albert EINSTEIN How I see the world Infinity Tile Victorian Tessellation Einstein Tessellation • Erno Rubik is the inventor of the Rubik's Cube, the most diabolical of all puzzles. Here is the type 6d base tile of Rubik's Cube: • Where is the border between good and evil (Bien et Mal)? Here is the type 1b base tile of Frontière (Boundary): • Here is the one of Nicolas Pavage 2, type 3a: • It is, of course, tempting to tessellate with its own name. Here is the type 1b base tile of Nicolas : • Finally with words, here is again "Infini" (Infinity) written in a manner that with its transformation by rotation fills exactly a rectangle. We can then assemble the words like bricks. • Then the same word in a parallelogram: Rubik's Cube Boundary Nicolas tessellation Nicolas Nicolas 2 colors Infinity Square Infinity Cube 8 DISTORTING THE STRUCTURES Return to summary What is worth to be done is worth to be well done.Nicolas POUSSIN The harmony of proportions satisfies the senses. Saint Thomas d'AQUIN hat is better is the enemy of what is good. But the good is often the friend of what is mediocrity! Let us try to find the better by distorting the structures for dress up concentric circles, spirals, perspective or volume effects. This will allow us to discover new horizons full of wonderful harmonies. Opposite, here is for example what produce a type 2S structure on a perspective: • Here is the tessellation (type 2Gb) of Cascade: • Here is the one of Miel (Honey), type 2Sb: • Here is the one of Plouf ! (splash!) type 1Sb: • The one of Spirale d'infini (Spiral of infinity), type 2d: Cascade Honey Splash! Spiral of Infinity Inversion 2 Möbius Ribbon Translation in the Infinity Einstein Head in the Stars Majax Star Tessellation The Fire Dance • Opposite here is the original tessellation of type 6b which served for Cercles (Circles). Nine colors are required so as not to have intersecting circles of lizards. • This same tessellation also served for the Icosaèdre aux lézards (Icosahedron with Lizards) below. An icosahedron is a polyhedron with twenty equilateral triangles for faces. Six colors are necessary to obtain a harmonious distribution. There are five lizards' noses by the vertex. • How to put seven lizards' noses by vertex? It is impossible on the normal plane, but not on a hyperbolic disk. The lizards are then reduced to infinity on a circular limit. To ensure the same color to the lizards which follow each other and that two columns of the same color do not intersect, seven colors are needed. In fact, in the disk, if one considers the colors, there are not two identical lizards. Circles Icosahedron with Lizards Circular Limit 9 PLAYING WITH ISOMETRIES Return to summary It is trust in life, to compete with the impossible.Panait ISTRATI To reach the inaccessible star, this is my quest. Jacques BREL e cannot go beyond the limits of the possible, but we can push them forward. To do this, what can we do? For example, have we the ability to alter or add isometries on basic polygons? The answer is yes. We then get super-tiles with amazing features. • Let us take the 3a base polygon: Let us add the isometries of the base polygon 1Gb, that is to say two glide reflections. We then get the next polygon: On a rhomb we have two different isometries of rotation order 3. Let us draw a motif according to this polygon. For example this "Li'l Wolf" opposite. He looks astounded because he comes to realize that he could tessellate the plane not only according to types 3a and 1Gb, but also in many other ways. Type 3a tessellation: Tile type 2c divided into two identical motifs Tile type 2Ge divided into two identical motifs Tile type 6c divided into two identical motifs Type 1Gb tessellation: Tile type 3b divided into three identical motifs Tile type 1a divided into four identical motifs Rose with Li'l Wolves • Now let us take the 6a base polygon. There we have an equilateral triangle with an isometry of rotation order 6 and one of rotation order 2: For example this fish: From this fish, we can, of course, make a tessellation of type 6a: Then draw a pattern so that each side of the rotation order 2 be the reduction by half of each compensated deformation of the rotation order 6: But we can also join to him three smaller identical fishes: In this way, this gives us a type 6d tile divided into four: But that is not all. Let us take again our fish and let him be rotated 6 times. We can add all around a ring of smaller fishes. Then again, add a ring of even smaller fishes, and so on to infinity. We then get a perfectly hexagonal figure. Hexagonal Limit • In a way reminiscent of the example above, but with a different result, let us take a kite of type 6c: Type 6c tessellation: Let us draw a motif so that both sides of the order 3 rotation are equal to the sides of the order 6 rotation by the tangent of 30° (0.5774). For example this butterfly: Type 6d tessellation: We can make a tessellation of type 6c but also of type 6d If we add four small butterflies as below: To construct the Butterfly flake below, we have to make an order 6 rotation. And then, on the periphery, add a ring of small butterflies four times more numerous. Then to this new periphery, make the same three times.We can, of course, continue to add rings to infinity. Each new ring then, will generate more and more points of infinity as below. All in all... we get an infinity of infinities. Butterfly flake • As for the butterfly, but reversing the sides of the order 3 rotation, we can make this bird:Here it is below by limiting to the fifth crown • Let us take the 4S base polygon, that is to say a square with two symmetrical rotations of order 4. Let us add it four rotations of order 2: Let us draw, for example, this bird: We can tessellate the plane according to different types or make a rose or a non-periodic tessellation such the lowest one. We see a teeming mass of birds amongst which some go right their way and a few rarer, illuminate the four horizons. Birds flake Cardinal Points • Let us see if we can draw a motif which has several base tiles: This "Good Doggie" will do the trick: Type 3b tessellation: Type 2Gg tessellation: Type 1Gc tessellation: Rose : • And if we find again our lizard friends. In the Smaller and Smaller print (N°35), Escher has used an isosceles right triangle of type 4a to reduce the lizards to infinity according to the diagram below: This allows us to draw two lizards by triangle instead of one: We can now make a order 4 rotation and reduce the lizards to infinity. The most direct way results an octogon. In fact, there is not only a couple of different lizards, but four. This allows adaptations.Escher, in Smaller and Smaller, drew nine different lizards. Let us invert the direction of triangles as follows: Octogonal limit Lizards flake Double limit I hope you had lots of fun watching this method. This will be the final word: FIN (the END in French). Return to summary Nota bene :If you liked this method, a little word in the "Guestbook"would make me happy.