Stated below are a few challenging problems. If you are first to publish a solution in a refereed journal, let me know, and collect your reward! Or, if you find a short solution and you are quite sure it is correct and complete, send it to ck6@evansville.edu. If accepted, your proof will be published on this site - see, for example, Problem 8. Note (added January 15, 2025): Because of the high cost of sending international money orders (etc.), payments for solutions after January 1, 2025 will be as donations that Clark Kimberling will make in your name to Online Encyclopedia of Integer Sequences: Donations This sequence is identical to its own runlength sequence. Reward: $200.00 for publishing a solution of any one of the five problems stated in Integer Sequences and Arrays. For many years, the sequence was thought to originate as indicated here: William Kolakoski, "Self generating runs, Problem 5304," American Mathematical Monthly 72 (1965) 674. For a proof that the Kolakoski sequence is not periodic, see the same Monthly 73 (1966) 681-682. However, it quite clearly occured earlier and can now be called the Oldenburger-Kolakoski sequence. It was discussed by Rufus Oldenburger, "Exponent trajectories in symbolic dynamics", Transactions of the American Mathematical Society 46 (1939), 453-466. See also Kolakoski Sequence at MathWorld and Sequence A000002 at the Online Encyclopedia of Integer Sequences. 2. A Sequence Is every positive integer a term of this sequence: 1, 3, 5, 4, 10, 7, 15, 8, 20, 9, 18, 24, 31, 14, 28, 22, 42, 35, 33, 46, . . . ? Reward: $300.00. To generate the sequence, visit Kimberling Sequence at MathWorld and Generator. For a discussion and variant of the problem, see Richard K. Guy, Unsolved Problems in Number Theory, second edition, Springer-Verlag, 1994. The problem originates in C. Kimberling, Problem 1615, Crux Mathematicorum 17 (1991) 44. 3. Repetition-resistant Sequence Reward: $100.00 (Paid). SOLVED by Alejandro Dau, February 2003 Consider the sequence (or infinite word) R = (r1, r2, r3, . . .) = 0100011010111001001110110000101... (commas deleted). The first segment of length 1 to repeat is "0", at r1 and r3. The first segment of length 2 to repeat is "00", at r5. The first segment of length 3 to repeat is "010", at r10. The sequence R is constructed to avoid repetitions "as long as possible", as follows: rn+1 = 1 if and only if (r1, r2, . . . , rn, 0), but not (r1, r2, . . . , rn, 1), has greater maximal repeated segment length than (r1, r2, . . . , rn) has. A finite string of 0's and 1's is called a word. Does every word occur in R? If so, then it is easy to see that every word repeats infinitely many times in R, which is notable, since the rule for generating R tries to resist repetition. This problem originates, in more general form, in C. Kimberling, Problem 2289, Crux Mathematicorum 23 (1997) 501; [no solutions received: 24 (1998) 525]. The problem has been solved in the affirmative: the binary word R does indeed contain every binary word infinitely many times. The word R corresponds to a choice-sequence consisting of all 0's. Any binary sequence can serve as a choice-sequence; details are given in the reference cited just above. The solution appears in Crux Mathematicorum 29 (2003) 320-321. 4. A Hard Count Reward: $100.00 Like the theory of relativity, this problem has a special version and a general version. The special case originates in C. Kimberling, Problem 2386, Crux Mathematicorum 24 (1998) 426 and is equivalent to the following: Write "1". Skip some space and count the one 1 you've written so far by writing "1 1". Skip some more space and count the three 1's you've written so far by writing "3 1" - except write it with 3 above 1, like this: 3 1. Skip some space, and count the four 1's and one 3 that you've written so far by writing 4 1 1 3. Skip some space, and count what you've written so far by writing 6 2 1 1 3 4. Skip some space, and count what you've written so far by writing 8 1 3 2 1 1 2 3 4 6. If this procedure continues indefinitely, will every positive integer eventually be written? Now for the general form of this problem. It's just like the special, except that we start with an arbitrary initial counting; that is, instead of starting with one 1, start with a(1) a(2) a(3) . . . a(n) b(1) b(2) b(3) . . . b(n), where all the a(i) and b(i) are distinct. The problem is to prove or disprove that every positive integer will eventually be written during the counting procedure. 5. The MD Problem (M=Multiply, D=Divide) Reward: $100.00 (Paid). SOLVED by Mateusz Kwasnicki, January 2004 Let a1 = 1, and for n > 1, define an = [an-1/2] if this number is not in the set {0, a1, . . . , an-1}, and an = 3an-1 otherwise. Does every positive integer occur exactly once in this sequence? The problem originates in C. Kimberling, Problem 2248, Crux Mathematicorum 26 (2000) 238; [no solutions received: 27 (2001) 345]. In the statement of the problem, the notation [x] means "the greatest integer that is n) is generated by repeatedly multiplying by 3 and dividing by 2. The MD sequence generated above begins like this: 1, 3, 9, 4, 2, 6, 18, 54, 27, 13, 39, 19, 57, 28, 14, 7, 21, . . . You can obtain another MD sequence by using 2 as multiplier and 3 as divisor: 1, 2, 4, 8, 16, 5, 10, 3, 6, 12, 24, 48, 96, 32, 64, 21, 7, 14, 28, 9, 18, 36, . . . Or, let m and d be any two relatively prime integers greater than 1. Does the resulting MD sequence contain every positive integer exactly once? Mateusz Kwasnicki of Wroclaw University of Technology solved the MD problem in general and in the affirmative. See Crux Mathematicorum 30 (2004) 235-239. 6. Are They All Even? Reward: $50.00 (Paid) SOLVED by Michael Behrend, December 2010 Begin an array by writing the Fibonacci numbers 1, 2, 3, 5, 8, ... Start row 2 with the least unused positive integer, which is 4; follow 4 by 6, and finish the row using the Fibonacci recurrence (i.e., add the two most recent numbers to produce the next, so that row 2 starts with 4, 6, 10, 16, 26, 42). Start row 3 with the least unused, which is 7, follow 7 by 12, and then use the recurrence to produce 19, 31,.... Continue in this manner (the rule for the second term in each new row is revealed below), getting rows 1 to 4 starting like this: 1 2 3 5 8 13 . . . 4 6 10 16 26 42 . . . 7 12 19 31 50 81 . . . 9 14 23 37 60 97 . . . . . Here is the recipe for the second number in each row: let r be the golden ratio, (1+sqrt(5))/2, let i be the number of the row, and let x be the first number in the row; then the second number is [rx] if i is even, and it is [rx]+1 if i is odd. For example, row 5 starts with the least unused, which is x=11, and this is followed by [11r]+1, which is 18. Is every number in column 2 even? In anticipation that the answer is yes, the array is introduced as the Even Second Column Array in C. Kimberling, "The First Column of an Interspersion," The Fibonacci Quarterly 32 (1994) 301-315. Michael Behrend proved that the answer is yes: Proof that they are all even. 7. The Mysterious B Sequence Reward: $75.00 (Paid - as a contribution to the Online Encyclopedia of Integer Sequences) SOLVED by Peter Kosinar, February 2011 For any sequence A = (a(0), a(1), a(2), . . .) of positive real numbers, create a sequence B from A as follows: let b(0) = a(0) and for k>0, let U = [a(2k-1)]/2, V = a(2k), W = 4b(k-1), b(k) = V - U/W. (Assume for each k that W is not zero.) For a couple of easy examples, start with A = (1, 2, 3, 4, . . .) and A = (1, 1, 1, 1, . . .). Here's the unsolved problem: determine, with proof, conditions on c and d for which the arithmetic sequence A = (c, c + d, c + 2d, c + 3d, . . .) yields b(k) > 0 for every k. Peter Kosinar found a necessary and sufficient condition to be 0 < d < c, then the sequence B contains one and only one negative number: On The Mysterious B Sequence. For an associated array of numbers, see A186158 at the Online Encyclopedia of Integer Sequences. 8. Is This Number Irrational? Reward: $50.00 (Paid). SOLVED by Matthew Albano, June 2010 There is a unique shape of triangle ABC that is both side-golden and angle-golden (in contrast to a single shape of rectangle that is called the golden rectangle). The angles of ABC are B, tB and π - B - tB, where t is the golden ratio. The terms "side-golden" and "angle-golden" refer to partitionings of ABC, each in a manner that matches the continued fraction [1, 1, 1, . . .] of t. The special number B is the number between 0 and π such that sin(tB) = t sin(B). The number B is 0.65740548297653229803685415259396854512654145994648783937, which, as an angle, is between 37 and 38 degrees. Can you prove, or disprove, that B is irrational? Reference: A152149 in the Online Encyclopedia of Integer Sequences. Matthew Albano proved that this is an easy consequence of the Lindemann-Weierstrass theorem: that if x(0), x(1), . . . , x(m) are distinct algebraic numbers, then the numbers ex(0), ex(1), . . . , ex(m) are linearly independent over the algebraic numbers. (See, for example, Theorem 3.4, page 44, of Making Transcendence Transparent: an Intuitive Approach to Classical Transcendental Number Theory, by Edward B. Burger and Robert Tubbs, Springer, 2004.) Write sin(tB) - t sin B = 0, and suppose that B is algebraic. The identity sin A = (eiA - e-iA)/2i then gives eitB - e-itB - teiB + te-iB = 0. Since the coefficients and exponents are algebraic, this equation implies, by the Lindemann-Weierstrass theorem, that the numbers itB, -itB, iB, and -iB are not distinct. This contradiction proves that B is transcendental. 9. A Swappage Problem Reward: $25.00 (Paid) SOLVED by Vincent Russo and Loren Schwiebert, 2010 Let L = (1, 3, 4, 6, 8, . . .) be the lower Wythoff sequence, A000201, and let U be the complement of L; i.e., U = (2, 5, 7, 10, . . .), the upper Wythoff sequence, A001950. For each odd U(n), let L(m) be the least number in L such that U(n) < L(m) and swap U(n) and L(m). The resulting sequence, called a "swappage" of L, is V = (2, 4, 6, 10, 11, 12, 14, 18, 19, 20, 26, . . .) = A141104. Let S(n) = (1/2)V(n) for every n. Is the complement of S (in the set of nonnegative integers) the set of numbers that comprise the sequence A004976 ? Russo and Schwiebert prove that the answer is yes in their article, "Beatty Sequences, Fibonacci Numbers, and the Golden Ratio," The Fibonacci Quarterly49 (2011) 151-154. 10. Curve Closest to Sphere Reward: $50.00 or $100.00 Let S be given by x^2+y^2+z^2 = 1. Suppose that C is a simple closed curve on S. For any point P on S, define the distance from P to C to be the minimal arclength from P to Q as Q goes around C. Find parametric equations for such a curve C of length 4π that minimizes the mean distance from S to C; that is, the mean distance taken over all points P on S. The solution must include proof of minimization. Can you solve this problem with arbitrary L > 2π in place of 4π? Are there in the literature precedent for this problem. A few closed simple spherical curves (which probably do not minimize the mean distance) can be viewed at Gallery of Space Curves Made from Circles and Gallery of Bishop Curves and Other Spherical Curves. 11. Run-length Sequences Reward: $75.00 For any sequence s consisting of 1's and 2's, let r(s) denote the length of the nth run of same symbols in s. There is a unique nontrivial sequence s such that s(1) = 1 and r(s(n)) = s(n) for all n. Successive terms of s and r, easily generated, are shown here: s = (1, 1, 2, 1, 1, 2, 2, 1, 2, 2, 1, 1, 2, 1, 1, 2, 2, 1, 2, 1, 1, 2, 2, . . .) r(s) = (2, 1, 2, 2, 1, 2, 1, 1, 2, 1, 1, 2, 2, 1, 2, 1, 1, . . .). Here's the problem: prove or disprove that every finite sequence of 1's and 2's occurs in s. For example, the initial segment 1121 of s occurs in s beginning at the 14th term. This problem first appeared as Problem 90 in Mathematicsche Semesterberichte 44 (1997) 94-95. More terms are given at A025142 and A025143. 12. Prime Separator Array Reward: $25.00 First, decree that T(1,1) = 1. Then for n > 0, let S(n) = {(i,j) : i and let T(1,n+1) = least positive integer not among T(i,j) for (i,j) in S(n) and not (T(n+1); T(m,n+1) = T(m,1)*T(1,n+1). These rules generate an array T which starts out like this: 1 2 4 7 9 13 15 18 23 25 29 3 6 12 21 27 39 45 54 69 75 87 5 10 20 35 45 65 75 90 115 125 145 8 16 32 56 72 104 120 144 184 200 232 11 22 44 77 99 143 165 198 253 275 319 Every prime is in row 1 of T or column 1, but not both. The difference sequence of row 1 starts with 1,2,3,2,4,2,3,5,2,4,2,4,5,4,2,4,7,2,3,2,4,3,5,4,5,... Here's the problem: prove (or disprove) that this difference sequence is bounded. More terms of the array and its first row are given at A129258 and A129259. 13. All the Positive Integers and Also All the Integers? Reward: $100.00 (see below) This problem involves an algorithm the "self-generates" a sequence of nonnegative integers and a sequence of integers. First, some notation: let a(1) = 1 and d(1) = 0. For k > 0, let x = a(k), and let P(k) = {a(1), a(2), . . . , a(k)}, and D(k) = {d(1), d(2), . . . , d(k)}. Step 1. If there is a negative integer h in D(k) such that x + h is not in P(k), and x > 0, then let d(k+1) be the greatest such h, let a(k+1) = x + h, and return to Step 0. Else Step 2. Let h be the least positive integer not in D(k) such that x + h is not in P(k). Let a(k+1) = x + h and d(k+1) = h, and do Step 1. Problems and Rewards: $25 for proof (or $20 for counterexample) of each of the following propositions. (1) a(k) runs through all the positive integers; (2) d(k) runs through all the integers; (3) if d(k) > 0, then d(k+1) > 0 or d(k+2) > 0 or d(k+3) > 0; (4) if d(k) < 0, then d(k+1) < 0 or d(k+2) < 0 or d(k+3) < 0. The sequence (a(k)) begins with 1,2,4,3,6,10,8,5,11,7,12,19,14,22,16,9, ... The sequence (d(k)) begins with 0,1,2,-1,3,4,-2,-3,6,-4,5,7,-5,8,-6,-7,9. More terms of the sequences are given at A131388 and A131389. 14. Never 3? Let G = (1 + 51/2)/2 and f(n) = floor(n2G) - n floor(nG), for n = 1,2,3,... . Examples: f(n) = 0 for n = 1, 2, 5, 13, 34, ... f(n) = 1 for n = 4, 10, 16, 68, 178, ... f(n) = 2 for n = 3, 7, 18, 47, 123, ... Reward of $20 for a proof that f(n) is never 3. (Paid) SOLVED by Michael Behrend, December 2010 Michael Behrend proved that the answer is yes: Proof that they are all even. 15. Infinitely Many Primes in Every Row? Reward: $50.00 Consider the following array of all the natural numbers: 1 2 4 7 11 16 22 29 37 46 56 3 5 8 12 17 23 30 38 47 57 68 6 9 13 18 24 31 39 48 58 69 81 10 14 19 25 32 40 49 59 70 82 95 15 20 26 33 41 50 60 71 83 96 110 This array is discussed at A000027 and A185787. Prove or disprove that every row contains infinitely many primes. Note added (March 25 2012): the problem is hard. Charles Greathouse writes that it reduces to the Hardy-Littlewood Conjecture F...it's not hard to show that each column corresponds to an integer-valued polynomial satisfying the requirements of the conjecture (the discriminant is always negative and hence nonsquare); for total conformance to the conjecture as printed replace it with two polynomials, one for even columns and one for odd. "So," he writes, "I don't expect to see a proof of this one any time soon." Pedja Terzic calls attention to rows instead of columns: the nth term of the first row is given by an = (n2 - n + 2)/2, so that for n=2k, we have a2k = P(k)=2k2 - k + 1 and for n=2k-1, we have a2k-1 = Q(k)=2k2 - 3k + 2. Both P(k) and Q(k) are irreducible over the integers. Also, GCD(P(1), P(2),...)=1 and GCD(Q(1), Q(2),...)=1. Therefore, according to the Bunyakowsky conjecture, each of the sequences (P(k)) and (Q(k)) includes infinitely many primes. The same holds for all the other rows. So, one should prove or disprove the Bunyakowsky conjecture. 16. Primes in Beatty Sequences Reward: $0.00 (see note below) Prove or disprove that if r is an irrational number between 1 and 2, then there are infinitely many primes of the form floor(n*r). This problem is stated at A025142. Note added (March 25 2012): a proof is known. Michael Behrend notes that a proof can be found in I. M. Vinogradov, The Method of Trigonometrical Sums in the Theory of Numbers, Interscience Publishers, London and New York, 1954, page 180. There, Vinogradov writes that "the result [that if x is any fixed irrational number then the fractional parts of nx are uniformly distributed between 0 and 1] can be expressed in another form, as was suggested by Professor Heilbronn: [if x>1 and x is not an integer, the numbers [nx] include infinitely many primes]." 17. Special M Reward: $50.00 (Paid) Let r denote the golden ratio, (1+sqrt(5))/2, and let [ ] denote the floor function. For fixed n, let u(k)=[k*r^n], let v(k)=[k*r]^n, and let w(k)=[v(k)/k^(n-1)]. We can expect w to have about the same growth-rate as u. Prove or disprove that for every fixed n>0, as k ranges through all the positive integers, there is a number M such that u(k)-w(k) takes each of the values 1,2,...,M infinitely many times, and u(k)-w(k) Michael Behrend proved that M exists for r = (1+sqrt(5))/2 and that there are other value of r for which there is no such M. The solution: Special M 18. Triangles with Interlacing Rows Reward: $50.00 In how many ways can the numbers 1, 2, ..., n(n+1)/2 be arranged in a triangular format with interlacing rows? That is, if a(i,j) denotes the jth number in row i, then a(i,j) is between a(i+1,j) and a(i+1,j+1), as in the following examples (for n = 3): 19. Congruent Incircles Point Reward: $50.00 (paid) Noam Elkies proved that there is a point X in the plane of an arbitrary ABC such that the triangles AXB, BXC, CXA have congruent incircles. Find reasonable barycentric coordinates for X. (The point X is listed as X(5394) in the Encyclopedia of Triangle Centers: ETC - Part 3) A recent solution by Jeremy Tan is presented at X(5394). 20. Euler-Morley-Zhao Point Reward: $50.00 (Paid) SOLVED by Shi Yong, January 3, 2013 Let A'B'C' be the Morley equilateral triangle of an arbitary triangle ABC. Zhao Yong of Anhui, China, discovered that the Euler lines of the triangles A'BC, AB'C, ABC' concur in a point. Find (reasonable) barycentric coordinates. The point is X(5390) in the Encyclopedia of Triangle Centers: ETC - Part 3 Solution by Shi Yong, Shanghai, China, January 3, 2013 Let DEF be the Morley triangle of ABC, and label the vertices, A, B, C, D, E, F as shown in the following pictures: Picture 1 and Picture 2 At vertex F, place the origin of the complex plane, with axes scaled so that the foot of the altitude of F on side AB is the point 1+0*i. Write angles as u = BAC, v = CBA, w = ABC = π - u - v. Then a = - cos(2u/3) + i*sin(2u/3) and b = - cos(2v/3) - i sin(2v/3). Rotation, translation, and dilation yield A = 2/(a + 1), B = 2/(b + 1), C = 2(a + b - 1)/(a2 + ab + b2), as shown in Picture 1. Let w = (1 + 31/2i)/2. P1 = a3b3 + a2b4 + ab4 + a4 - a3b - ab3 + b4 + a2b + b3 + ab Q1 = (a - b)(a3b2 + a2b3 + a3b + a2b2 + ab3 + a2 + b2) P2 = ab2 + a2 - 3ab + b2 + b Q2 = (a - b)(ab + 1) The point Z = X(5390), shown as Zhao in Picture 1 and as X(5390) in Picture 2, is given by Z = 2(Q1 + wP1)/(Q2 + wP2)/((a + 1)(b + 1)(a2 + ab + b2)) Define A' by applying the transformations a -> 1/a, b -> 1/b, c -> 1/c in the formula for A. Likewise, define B' from B, C' from C, P'1 from P1, Q'1 from Q1, P'2 from P2, and Q'2 from Q2. It will be convenient to abbreviate X(5390) as Z. Define Z' by applying to Z the transformations a -> 1/a, b -> 1/b, c -> 1/c, P1 -> P'1, Q1 -> Q'1, P2 -> P'2, Q2 -> Q'2. Let x : y : z be barycentric coordinates for Z. Solving the system Ax + By + Cz = Z, A'x + B'y + C'z = Z', x + y + z = 1 gives x = (a + 1)(Zb3 - 2b2 + 2b - Z')/(2(b - a)(b2 - b + 1)) y = (b + 1)(Za3 - 2a2 + 2a - Z')/(2(a - b)(a2 - a + 1)) z = (a2 + ab + b2)(2 - Z - Z')/(2(a2 - a + 1)(b2 - b + 1)) With a computer algebra system, it can be shown that the complex conjugates of x, y, z are equal to x, y, z, respectively; that is, they are real-valued. 21. Jump Sequences Reward: $50.00 (Paid) For fixed positive integer m, let a(n) be the increasing sequence of nonnegative integers k such that round(k1/m) < round((k+1)1/m). Prove or disprove that a(n) is a homogeneous linear recurrence sequence. Example: for m = 3, see sequence A219085 in the Online Encyclopedia of Integer Sequences SOLVED by David Moews, January 2013 David Moews proved for each m > 0, the sequence a(n) is a linear recurrence sequence (LRS), because it is the sum of a polynomial and a periodic sequence, and both of these are LRSs. A proof follows. Starting with the formula in the Comments section of A219085, floor((n + 1/2)m) = (n + 1/2)m - frac((n + 1/2)m) = (n + 1/2)m - 2m((2n + 1)m mod 2m), where frac(x) := x - floor(x) is the fractional part of x, and (a mod b) := b frac(a/b). The sequence (n + 1/2)m is a polynomial of degree m in n, so it is annihilated by (E - 1)m+1, where E is the shift operator. The sequence (2n + 1)m mod 2m depends only on n mod 2m-1, so it is annihilated by E2m-1 - 1. Therefore the sequence floor((n + 1/2) is annihilated by (E - 1)m(E2m-1 - 1), so it is a homogeneous LRS of order at most m + 2m-1. If m is even the upper bound on the order can be lowered. Suppose that m is divisible by 2k for some k > 0. Then (2n + 1)m mod 2m depends only on n mod 2m-k-1, so that the sequence frac((n + 1/2)m) is annihilated by E2m-k-1 - 1. Also, the value of (2n + 1)m mod 2m is unchanged if n is replaced by 2m-k-1 - n - 1. If m > 3, this replacement interchanges even and odd values of n, so that the sum of the residues coming from even values of n equals that coming from odd values. This shows that the sequence frac((n + 1/2)m) is annihilated by 1 - E + E2 - E3 + ... - E2m-k-1 - 1 = (1 - E2m-k-1)/(1 + E). Therefore, if m is even, the sequence floor((n + 1/2)m) is a homogeneous LRS of order at most m + 2m-k-1 - 1. In the special call m = 2, this order is at most 3. 22. Lucas and Zeckendorf Representations Reward: $30.00 (Paid) Let U(n) and V(n) be the number terms in the Lucas and Zeckendorf representations, respectively, of all the numbers 1,2,...,n. Prove or disprove that V(n) >= U(n) for all n and that V(n) = U(n) for infinitely many n. Michael Behrend proved both propositions: Lucas and Zeckendorf representations 23. Special Numbers Reward: $50.00 Characterize the numbers r for which the sequence floor(n*r) contains a homogeneous linearly recurrent subsequence. 24. Harmonic Limit Reward: $30.00 Let f(n) = 1/(H(n) - g - log n), where H(n) = 1+ 1/2 + 1/3 + ... + 1/n and g is the Euler-Mascheroni constant. Prove that lim(f(n) - 2n) = 1/3. SOLVED by Goudout �lie, August 2013 The asymptotic equation H(n) = g + log n + 1/(2n) - 1/(12n�) + o(1/n�) is proved, for example, in Graham, Knuth, and Patashnik, Concrete Mathematics, p. 466. Then 1/(H(n) - g - log n) = 2n/(1 - 1/(6n) + o(1/n)) = 2n*(1 + 1/(6n) + o(1/n)) for big values of n, so that 1/(H(n) - g - log n) = 2n + 1/3 + o(1) for big values of n. For connections to the joint ranking of the harmonic numbers and the numbers g + log k for k>=1, see A226894 at the Online Encyclopedia of Integer Sequences. Clark Kimberling Home Page Ring theory 06/11/2019 by Yu · Published 06/11/2019 · Last modified 06/13/2019 Prove that if R is a commutative ring and $\frakN(R)$ is its nilradical, then the zero is the only nilpotent element of $R/\frakN(R)$. That is, show that $\frakN(R/\frakN(R))=0$. Read solution Add to solve later Ring theory 06/10/2019 by Yu · Published 06/10/2019 Let R be a principal ideal domain. Let $a\in R$ be a nonzero, non-unit element. Show that the following are equivalent. (1) The ideal (a) generated by a is maximal. (2) The ideal (a) is prime. (3) The element a is irreducible. Read solution Add to solve later Ring theory 06/09/2019 by Yu · Published 06/09/2019 Let R be a finite commutative ring with identity 1. Prove that every prime ideal of R is a maximal ideal of R. Read solution Add to solve later Ring theory 12/06/2017 by Yu · Published 12/06/2017 Let R and R' be commutative rings and let $f:R\to R'$ be a ring homomorphism. Let I and I' be ideals of R and R', respectively. (a) Prove that $f(\sqrt{I}\,) \subset \sqrt{f(I)}$. (b) Prove that $\sqrt{f^{-1}(I')}=f^{-1}(\sqrt{I'})$ (c) Suppose that f is surjective and $\ker(f)\subset I$. Then prove that $f(\sqrt{I}\,) =\sqrt{f(I)}$ Read solution Add to solve later Ring theory 12/05/2017 by Yu · Published 12/05/2017 Let $I=(x, 2)$ and $J=(x, 3)$ be ideal in the ring $\Z[x]$. (a) Prove that $IJ=(x, 6)$. (b) Prove that the element $x\in IJ$ cannot be written as $x=f(x)g(x)$, where $f(x)\in I$ and $g(x)\in J$. Read solution Add to solve later Ring theory 12/01/2017 by Yu · Published 12/01/2017 Is it true that a set of nilpotent elements in a ring R is an ideal of R? If so, prove it. Otherwise give a counterexample. Read solution Add to solve later Ring theory 11/28/2017 by Yu · Published 11/28/2017 Let R be a commutative ring with 1 such that every element x in R is idempotent, that is, $x^2=x$. (Such a ring is called a Boolean ring.) (a) Prove that $x^n=x$ for any positive integer n. (b) Prove that R does not have a nonzero nilpotent element. Read solution Add to solve later Ring theory 11/28/2017 by Yu · Published 11/28/2017 Let R be a commutative ring with 1. Suppose that the localization $R_{\mathfrak{p}}$ is a Noetherian ring for every prime ideal $\mathfrak{p}$ of R. Is it true that A is also a Noetherian ring? Read solution Add to solve later Ring theory 11/24/2017 by Yu · Published 11/24/2017 Let R be a ring and assume that whenever $ab=ca$ for some elements $a, b, c\in R$, we have $b=c$. Then prove that R is a commutative ring. Read solution Add to solve later Ring theory 11/02/2017 by Yu · Published 11/02/2017 Let R be a commutative ring with 1. Prove that if every proper ideal of R is a prime ideal, then R is a field. Read solution Add to solve later Ring theory 10/19/2017 by Yu · Published 10/19/2017 Let R be an integral domain and let I be an ideal of R. Is the quotient ring R/I an integral domain? Read solution Add to solve later Ring theory 09/27/2017 by Yu · Published 09/27/2017 Let $\Z[x]$ be the ring of polynomials with integer coefficients. Prove that \[I=\{f(x)\in \Z[x] \mid f(-2)=0\}\] is a prime ideal of $\Z[x]$. Is I a maximal ideal of $\Z[x]$? Read solution Add to solve later Ring theory 08/20/2017 by Yu · Published 08/20/2017 Let R be a ring with 1. Suppose that a, b are elements in R such that \[ab=1 \text{ and } baeq 1.\] (a) Prove that $1-ba$ is idempotent. (b) Prove that $b^n(1-ba)$ is nilpotent for each positive integer n. (c) Prove that the ring R has infinitely many nilpotent elements. Read solution Add to solve later Ring theory 08/18/2017 by Yu · Published 08/18/2017 Let R be a ring with $1eq 0$. Let $a, b\in R$ such that $ab=1$. (a) Prove that if a is not a zero divisor, then $ba=1$. (b) Prove that if b is not a zero divisor, then $ba=1$. Read solution Add to solve later Ring theory 08/11/2017 by Yu · Published 08/11/2017 Let R and S be rings with $1eq 0$. Prove that every ideal of the direct product $R\times S$ is of the form $I\times J$, where I is an ideal of R, and J is an ideal of S. Read solution Add to solve later Ring theory 08/09/2017 by Yu · Published 08/09/2017 (a) Prove that every prime ideal of a Principal Ideal Domain (PID) is a maximal ideal. (b) Prove that a quotient ring of a PID by a prime ideal is a PID. Read solution Add to solve later Ring theory 08/08/2017 by Yu · Published 08/08/2017 · Last modified 08/09/2017 Let I be a nonzero ideal of the ring of Gaussian integers $\Z[i]$. Prove that the quotient ring $\Z[i]/I$ is finite. Read solution Add to solve later Ring theory 08/07/2017 by Yu · Published 08/07/2017 Let R and S be rings. Suppose that $f: R \to S$ is a surjective ring homomorphism. Prove that every image of an ideal of R under f is an ideal of S. Namely, prove that if I is an ideal of R, then $J=f(I)$ is an ideal of S. Read solution Add to solve later Ring theory 08/06/2017 by Yu · Published 08/06/2017 (a) Let F be a field. Show that F does not have a nonzero zero divisor. (b) Let R and S be nonzero rings with identities. Prove that the direct product $R\times S$ cannot be a field. Read solution Add to solve later Ring theory 08/05/2017 by Yu · Published 08/05/2017 Let R be a commutative ring with identity $1eq 0$. Suppose that for each element $a\in R$, there exists an integer $n > 1$ depending on a. Then prove that every prime ideal is a maximal ideal. Read solution Add to solve later Mathematics is more than just numbers—it's a universal tool driving innovation, problem-solving, and sustainable progress. From tackling climate change to powering artificial intelligence, math is essential to shape a better world. On this International Day of Mathematics, students are encouraged to explore how math supports global solutions and unlocks exciting careers in fields like cryptography, game development, and actuarial science, proving that math truly is the language of the futureMathematics is not just about solving equations—it's about solving real-world problems. With global challenges like climate change, health crises, and sustainability, mathematics plays a vital role in crafting effective, data-driven solutions. Take climate change, for instance. Mathematical modeling helps scientists predict temperature rise, sea level changes, and the impact of carbon emissions. Data analytics and statistics allow researchers to monitor pollution patterns and develop more sustainable practices. In pandemics, math has been crucial in forecasting infection spread, resource planning, and vaccine distribution. These applications demonstrate how math can literally save lives. Beyond the environment and health, mathematics also powers technological innovation. From algorithms that drive social media and search engines to the backbone of artificial intelligence (AI) and machine learning (ML), math is everywhere. Whether it's robotics, data science, or cryptography, mathematics underpins the logic and precision required for modern digital systems. For example, linear algebra fuels image recognition, while probability and statistics empower machines to "learn" from data. Importantly, this blend of math and technology is also reshaping career paths for young minds. While traditional math careers like teaching and research remain relevant, new-age professions are rapidly emerging. Students passionate about problem-solving can now pursue roles as actuaries, who assess risk in finance and insurance using complex models. Those with a creative edge may find their calling in game development, where physics engines and animation rely on mathematical calculations. For those drawn to cybersecurity, cryptography offers an exciting blend of logic, coding, and secrecy. The versatility of math means that it not only drives change—it creates opportunity. Students of today have the chance to use mathematics not just in theory but in ways that genuinely shape the world for the better. Welcome to the Open Problem Garden, a collection of unsolved problems in mathematics. Here you may: Read descriptions of open problems. Post comments on them. Create and edit open problems pages (please contact us and we will set you up an account. Unfortunately, the automatic process is too prone to spammers at this moment.) Help us Grow! We are eager to expand, so we are inviting contributions both large and small from all areas of mathematics. Many thanks to our contributors! More about the Garden Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. Group Theory 12/14/2017 by Yu · Published 12/14/2017 · Last modified 06/06/2019 Let G be a group of order 57. Assume that G is not a cyclic group. Then determine the number of elements in G of order 3. Read solution Add to solve later Group Theory 12/12/2017 by Yu · Published 12/12/2017 Let G be a group. Suppose that the number of elements in G of order 5 is 28. Determine the number of distinct subgroups of G of order 5. Read solution Add to solve later Group Theory 12/12/2017 by Yu · Published 12/12/2017 Let G be a group and let H_1, H_2 be subgroups of G such that $H_1 ot \subset H_2$ and $H_2 ot \subset H_1$. (a) Prove that the union $H_1 \cup H_2$ is never a subgroup in G. (b) Prove that a group cannot be written as the union of two proper subgroups. Read solution Add to solve later Group Theory 12/02/2017 by Yu · Published 12/02/2017 · Last modified 12/01/2017 Let G be a finite group and let N be a normal subgroup of G. Suppose that the order n of N is relatively prime to the index $|G:N|=m$. (a) Prove that $N=\{a\in G \mid a^n=e\}$. (b) Prove that $N=\{b^m \mid b\in G\}$. Read solution Add to solve later Group Theory 11/29/2017 by Yu · Published 11/29/2017 Group Theory 11/27/2017 by Yu · Published 11/27/2017 Suppose that p is a prime number greater than 3. Consider the multiplicative group $G=(\Zmod{p})^*$ of order $p-1$. (a) Prove that the set of squares $S=\{x^2\mid x\in G\}$ is a subgroup of the multiplicative group G. (b) Determine the index $[G : S]$. (c) Assume that $-1otin S$. Then prove that for each $a\in G$ we have either $a\in S$ or $-a\in S$. Read solution Add to solve later Group Theory 11/24/2017 by Yu · Published 11/24/2017 Let G be a finite group. Let S be the set of elements g such that $g^5=e$, where e is the identity element in the group G. Prove that the number of elements in S is odd. Read solution Add to solve later Group Theory 11/22/2017 by Yu · Published 11/22/2017 Let m and n be positive integers such that $m \mid n$. (a) Prove that the map $\phi:\Zmod{n} \to \Zmod{m}$ sending $a+n\Z$ to $a+m\Z$ for any $a\in \Z$ is well-defined. (b) Prove that ϕ is a group homomorphism. (c) Prove that ϕ is surjective. (d) Determine the group structure of the kernel of ϕ. Read solution Add to solve later Group Theory 10/27/2017 by Yu · Published 10/27/2017 Is it possible that each element of an infinite group has a finite order? If so, give an example. Otherwise, prove the non-existence of such a group. Read solution Add to solve later Group Theory 09/28/2017 by Yu · Published 09/28/2017 Let G be a finite group of order $2n$. Suppose that exactly a half of G consists of elements of order 2 and the rest forms a subgroup. Namely, suppose that $G=S\sqcup H$, where S is the set of all elements of order in G, and H is a subgroup of G. The cardinalities of S and H are both n. Then prove that H is an abelian normal subgroup of odd order. Read solution Add to solve later Group Theory 09/22/2017 by Yu · Published 09/22/2017 Group Theory 09/19/2017 by Yu · Published 09/19/2017 Group Theory 09/06/2017 by Yu · Published 09/06/2017 Let N be a normal subgroup of a group G. Suppose that G/N is an infinite cyclic group. Then prove that for each positive integer n, there exists a normal subgroup H of G of index n. Read solution Add to solve later Group Theory 09/03/2017 by Yu · Published 09/03/2017 · Last modified 12/27/2017 Let x, y be generators of a group G with relation \begin{align*} xy^2=y^3x,\tag{1}\\ yx^2=x^3y.\tag{2} \end{align*} Prove that G is the trivial group. Read solution Add to solve later Group Theory 08/21/2017 by Yu · Published 08/21/2017 · Last modified 12/27/2017 Let G a finite group and let H and K be two distinct Sylow p-group, where p is a prime number dividing the order $|G|$ of G. Prove that the product HK can never be a subgroup of the group G. Read solution Add to solve later Group Theory 07/29/2017 by Yu · Published 07/29/2017 Let G be a nilpotent group and let H be a proper subgroup of G. Then prove that $H \subsetneq N_G(H)$, where $N_G(H)$ is the normalizer of H in G. Read solution Add to solve later Group Theory 07/28/2017 by Yu · Published 07/28/2017 Let G be an abelian group and let H be the subset of G consisting of all elements of G of finite order. That is, \[H=\{ a\in G \mid \text{the order of a is finite}\}.\] Prove that H is a subgroup of G. Read solution Add to solve later Group Theory 07/15/2017 by Yu · Published 07/15/2017 · Last modified 07/30/2017 Let $(\Q, +)$ be the additive group of rational numbers and let $(\Q_{ > 0}, \times)$ be the multiplicative group of positive rational numbers. Prove that $(\Q, +)$ and $(\Q_{ > 0}, \times)$ are not isomorphic as groups. Read solution Add to solve later Group Theory 06/30/2017 by Yu · Published 06/30/2017 · Last modified 10/25/2017 Let G be an abelian group. Let a and b be elements in G of order m and n, respectively. Prove that there exists an element c in G such that the order of c is the least common multiple of m and n. Also determine whether the statement is true if G is a non-abelian group. Read solution Add to solve later Group Theory 06/27/2017 by Yu · Published 06/27/2017 · Last modified 09/28/2017 Prove that every finite group having more than two elements has a nontrivial automorphism. (Michigan State University, Abstract Algebra Qualifying Exam) Read solution Add to solve later Mathematics is one of the pillars of human development that has constantly been extending the bounds of knowledge and capability. A few problems within, over centuries, have gone unsolved and have puzzled some of the greatest minds. Such unsolved problems are not useless, but they hold the potential to unlock newer boundaries in science and technologies that would enhance our understanding of the universe. This article is a discussion of ten such unsolved problems, their purposes, and the outcome the solution would bring. Who knows? Maybe the solution to one of these problems is brought by you, who grabs the Nobel Prize in his or her category.Why It's Important to Solve These ProblemsThe reason these mathematical problems are so important is that quite often, the solution to one of them has huge implications for the concerned field. For instance, a breakthrough in number theory would immediately imply many things concerning cryptography, the backbone of secure communication in today's digital world. Similarly, solutions to problems on topology can influence how we understand the shape and structure of the universe. These problems constitute the ultimate frontiers of human knowledge—opening up whole new areas of enquiry, giving rise to technological innovation, and deepening our understanding of the natural world.The Top 10 Unsolved Mathematical Problems 1. The Riemann HypothesisExplanation: The Riemann Hypothesis is about the distribution of prime numbers. Primes are numbers greater than 1 that are only divisible by 1 and themselves. The hypothesis suggests a specific pattern in the distribution of these prime numbers, based on a mathematical function called the Riemann zeta function.The Riemann Hypothesis concerns the zeroes of the Riemann zeta function, defined as:\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}for complex numbers sss with real part greater than 1. It can also be extended to other values of sss via analytic continuation. The hypothesis states that all non-trivial zeroes of the zeta function have a real part equal to 1/2.Impact if Solved: Solving this would revolutionize number theory and cryptography, impacting everything from internet security to the fundamentals of mathematics.Current Progress: Mathematicians have verified the hypothesis for many zeros of the Riemann zeta function, but a general proof remains elusive.2. P vs NP ProblemExplanation: This problem asks whether every problem whose solution can be quickly verified by a computer can also be quickly solved by a computer. In simpler terms, it questions whether finding solutions is as easy as checking them.For example, given a Sudoku puzzle, verifying a solution is quick (polynomial time), but finding the solution might not be.Impact if Solved: A solution would transform fields like cryptography, optimization, and algorithm design, potentially making many currently intractable problems solvable.Current Progress: Despite significant efforts, the problem remains unsolved. Most experts believe P ≠ NP, but no proof has been found.3. Navier-Stokes Existence and SmoothnessExplanation: This problem involves understanding the behavior of fluids. The Navier-Stokes equations describe how fluids like water and air flow, but it's unclear whether solutions to these equations always exist and behave smoothly.The Navier-Stokes equations describe fluid motion and are given by:\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot abla) \mathbf{u} = -abla p + u \Delta \mathbf{u} + \mathbf{f} where u is the velocity field, p is the pressure, ν is the kinematic viscosity, and f is the external force.The problem is to show whether solutions always exist and remain smooth for all time in three dimensions.Impact if Solved: A solution could revolutionize fluid dynamics, leading to advancements in engineering, meteorology, and medical imaging.Current Progress: While partial results exist, a complete understanding of these equations continues to escape from mathematicians.4. Birch and Swinnerton-Dyer ConjectureExplanation: This conjecture relates to elliptic curves, which are mathematical objects with applications in number theory and cryptography. It suggests a deep connection between the number of rational points on an elliptic curve and a specific function associated with the curve.This conjecture involves elliptic curves, which are equations of the form:y^2 = x^3 + ax + b The conjecture posits a relationship between the number of rational points on an elliptic curve and the behavior of its L-function L(E,s)L(E, s)L(E,s) at s=1s = 1s=1.Impact if Solved: Solving this would advance our understanding of elliptic curves, with significant implications for number theory and cryptography.Current Progress: Some cases have been proved, but the general case remains unsolved.5. Hodge ConjectureExplanation: The Hodge Conjecture involves certain shapes called algebraic cycles on complex manifolds. It suggests that certain classes of these cycles are actually combinations of simpler, algebraic cycles. Mathematically, it deals with cohomology classes and their representation as sums of algebraic cycles.The general expression involving the Hodge decomposition is:H^n(X, \mathbb{C}) = \bigoplus_{p+q=n} H^{p,q}(X) Impact if Solved: A solution would advance algebraic geometry, impacting areas such as string theory and the classification of complex shapes.Current Progress: Some specific cases have been resolved, but a general proof remains out of reach.6. Yang-Mills Existence and Mass GapExplanation: This problem comes from theoretical physics and involves quantum field theory. It seeks to prove the existence of a theory that accurately describes fundamental forces and predicts a property called the mass gap.Yang-Mills theory is a cornerstone of particle physics, describing how fundamental forces work. D_\mu F^{\muu} = J^u where F μν is the field strength tensor, D μ is the covariant derivative, and J ν is the current. The problem is to prove that the Yang-Mills equations have solutions that exhibit a mass gap, meaning particles described by the theory have a positive mass.Impact if Solved: A solution would profoundly affect our understanding of particle physics, potentially leading to new discoveries in quantum mechanics and field theory.Current Progress: While physicists use Yang-Mills theory with great success, a rigorous mathematical proof is still missing.7. The Collatz ConjectureExplanation: This conjecture involves a simple process: take any positive integer, halve it if it's even, or triple it and add one if it's odd. Repeat this process, and the conjecture states that you'll always end up at 1, no matter what number you start with.Impact if Solved: A solution would deepen our understanding of number theory and iterative processes.Current Progress: Despite its simplicity, this problem has resisted solution, with extensive computational evidence supporting the conjecture but no proof.8. The Twin Prime ConjectureExplanation: The Twin Prime Conjecture is a famous problem in number theory that suggests there are infinitely many pairs of prime numbers p and p+2 that are both prime. These pairs are called "twin primes". For example, (3, 5), (11, 13), and (17, 19) are all pairs of twin primes.Mathematically, a prime number p is a number greater than 1 that has no positive divisors other than 1 and itself. The conjecture is formally stated as:∃ primes (p,p+2) This means there are infinitely many pairs of primes p and p+2.Impact if Solved: Proving this would be a huge achievement in number theory, further illuminating the nature of primes.Current Progress: Recent advancements have shown that there are infinitely many prime pairs within a certain small gap, but the exact conjecture remains unproven.9. The Beal ConjectureExplanation: This conjecture is a generalization of Fermat's Last Theorem. It proposes that if A^x + B^y = C^z has solutions in positive integers A, B, C, x, y, and z where x, y, and z are all greater than 2, unless A, B, and C have a common prime factor.Impact if Solved: A solution would contribute to our understanding of Diophantine equations and number theory.Current Progress: Despite extensive evidence supporting the conjecture, it remains unproven, though it has been verified for many specific cases.10. The Erdős-Straus ConjectureExplanation: The Erdős–Straus Conjecture is a problem in number theory related to Egyptian fractions. An Egyptian fraction is a sum of distinct unit fractions, which are fractions with a numerator of 1. The conjecture states that for any integer n>1, the equation:\frac{4}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z} has positive integer solutions for x, y and z. In simpler terms, it suggests that the fraction 4n\frac{4}{n}n4 can always be expressed as the sum of three unit fractions.For example:For n=2, we have 4/2 = 2, and indeed, 2= \frac{1}{1} + \frac{1}{2} + \frac{1}{2} For n=3, we have \frac{4}{3} = \frac{1}{1} + \frac{1}{4} + \frac{1}{12} For n=5, we have \frac{4}{5} = \frac{1}{2} + \frac{1}{4} + \frac{1}{20} Impact if Solved: A solution would contribute to our understanding of Egyptian fractions and number theory.Current Progress: Despite significant numerical evidence supporting the conjecture, no general proof has been found.ConclusionThese ten unsolved mathematical problems represent some of the most challenging and intriguing puzzles in the field. Their solutions have the potential to unlock new knowledge and drive significant advancements in various areas of science and technology. While these problems are difficult, they also offer a unique opportunity for anyone with the curiosity and dedication to tackle them. Perhaps one of you reading this article will find the breakthrough needed to solve one of these problems and secure your place in history—and maybe even a Nobel Prize! Mathematical Reviews Norwegian Register for Scientific Journals and Series zbMATH This is a dynamic list and may never be able to satisfy particular standards for completeness. You can help by adding missing items with reliable sources. Many mathematical problems have been stated but not yet solved. These problems come from many areas of mathematics, such as theoretical physics, computer science, algebra, analysis, combinatorics, algebraic, differential, discrete and Euclidean geometries, graph theory, group theory, model theory, number theory, set theory, Ramsey theory, dynamical systems, and partial differential equations. Some problems belong to more than one discipline and are studied using techniques from different areas. Prizes are often awarded for the solution to a long-standing problem, and some lists of unsolved problems, such as the Millennium Prize Problems, receive considerable attention. This list is a composite of notable unsolved problems mentioned in previously published lists, including but not limited to lists considered authoritative, and the problems listed here vary widely in both difficulty and importance. Various mathematicians and organizations have published and promoted lists of unsolved mathematical problems. In some cases, the lists have been associated with prizes for the discoverers of solutions. List Number ofproblems Number unsolved or incompletely solved Proposed by Proposedin Hilbert's problems[1] 23 15 David Hilbert 1900 Landau's problems[2] 4 4 Edmund Landau 1912 Taniyama's problems[3] 36 – Yutaka Taniyama 1955 Thurston's 24 questions[4][5] 24 2 William Thurston 1982 Smale's problems 18 14 Stephen Smale 1998 Millennium Prize Problems 7 6[6] Clay Mathematics Institute 2000 Simon problems 15 <12[7][8] Barry Simon 2000 Unsolved Problems on Mathematics for the 21st Century[9] 22 – Jair Minoro Abe, Shotaro Tanaka 2001 DARPA's math challenges[10][11] 23 – DARPA 2007 Erdős's problems[12] > 934 617 Paul Erdős Over six decades of Erdős' career, from the 1930s to 1990s The Riemann zeta function, subject of the Riemann hypothesis[13] Of the original seven Millennium Prize Problems listed by the Clay Mathematics Institute in 2000, six remain unsolved to date:[6] Birch and Swinnerton-Dyer conjecture Hodge conjecture Navier–Stokes existence and smoothness P versus NP Riemann hypothesis Yang–Mills existence and mass gap The seventh problem, the Poincaré conjecture, was solved by Grigori Perelman in 2003.[14] However, a generalization called the smooth four-dimensional Poincaré conjecture—that is, whether a four-dimensional topological sphere can have two or more inequivalent smooth structures—is unsolved.[15] The Kourovka Notebook (Russian: Коуровская тетрадь) is a collection of unsolved problems in group theory, first published in 1965 and updated many times since.[16] The Sverdlovsk Notebook (Russian: Свердловская тетрадь) is a collection of unsolved problems in semigroup theory, first published in 1969 and updated every 2 to 4 years since.[17][18][19] The Dniester Notebook (Russian: Днестровская тетрадь) lists several hundred unsolved problems in algebra, particularly ring theory and modulus theory.[20][21] The Erlagol Notebook (Russian: Эрлагольская тетрадь) lists unsolved problems in algebra and model theory.[22] Main article: Algebra In the Bloch sphere representation of a qubit, a SIC-POVM forms a regular tetrahedron. Zauner conjectured that analogous structures exist in complex Hilbert spaces of all finite dimensions. Birch–Tate conjecture on the relation between the order of the center of the Steinberg group of the ring of integers of a number field to the field's Dedekind zeta function. Bombieri–Lang conjectures on densities of rational points of algebraic surfaces and algebraic varieties defined on number fields and their field extensions. Connes embedding problem in Von Neumann algebra theory Crouzeix's conjecture: the matrix norm of a complex function f {\displaystyle f} applied to a complex matrix A {\displaystyle A} is at most twice the supremum of | f (z) | {\displaystyle |f(z)|} over the field of values of A {\displaystyle A} . Determinantal conjecture on the determinant of the sum of two normal matrices. Eilenberg–Ganea conjecture: a group with cohomological dimension 2 also has a 2-dimensional Eilenberg–MacLane space K (G , 1) {\displaystyle K(G,1)} . Farrell–Jones conjecture on whether certain assembly maps are isomorphisms. Bost conjecture: a specific case of the Farrell–Jones conjecture Finite lattice representation problem: is every finite lattice isomorphic to the congruence lattice of some finite algebra?[23] Goncharov conjecture on the cohomology of certain motivic complexes. Green's conjecture: the Clifford index of a non-hyperelliptic curve is determined by the extent to which it, as a canonical curve, has linear syzygies. Grothendieck–Katz p-curvature conjecture: a conjectured local–global principle for linear ordinary differential equations. Hadamard conjecture: for every positive integer k {\displaystyle k} , a Hadamard matrix of order 4 k {\displaystyle 4k} exists. Williamson conjecture: the problem of finding Williamson matrices, which can be used to construct Hadamard matrices. Hadamard's maximal determinant problem: what is the largest determinant of a matrix with entries all equal to 1 or −1? Hilbert's fifteenth problem: put Schubert calculus on a rigorous foundation. Hilbert's sixteenth problem: what are the possible configurations of the connected components of M-curves? Homological conjectures in commutative algebra Jacobson's conjecture: the intersection of all powers of the Jacobson radical of a left-and-right Noetherian ring is precisely 0. Kaplansky's conjectures Köthe conjecture: if a ring has no nil ideal other than { 0 } {\displaystyle \{0\}} , then it has no nil one-sided ideal other than { 0 } {\displaystyle \{0\}} . Monomial conjecture on Noetherian local rings Existence of perfect cuboids and associated cuboid conjectures Pierce–Birkhoff conjecture: every piecewise-polynomial f : R m → R {\displaystyle f:\mathbb {R} ^{m}\rightarrow \mathbb {R} } is the maximum of a finite set of minimums of finite collections of polynomials. Rota's basis conjecture: for matroids of rank n {\displaystyle n} with n {\displaystyle n} disjoint bases B i {\displaystyle B_{i}} , it is possible to create an n × n {\displaystyle n\times n} matrix whose rows are B i {\displaystyle B_{i}} and whose columns are also bases. Serre's conjecture II: if G {\displaystyle G} is a simply connected semisimple algebraic group over a perfect field of cohomological dimension at most 2 {\displaystyle 2} , then the Galois cohomology set H 1 (F , G) {\displaystyle H^{1}(F,G)} is zero. Serre's positivity conjecture that if R {\displaystyle R} is a commutative regular local ring, and P , Q {\displaystyle P,Q} are prime ideals of R {\displaystyle R} , then dim ⁡ (R / P) + dim ⁡ (R / Q) = dim ⁡ (R) {\displaystyle \dim(R/P)+\dim(R/Q)=\dim(R)} implies χ (R / P , R / Q) > 0 {\displaystyle \chi (R/P,R/Q)>0} . Uniform boundedness conjecture for rational points: do algebraic curves of genus g ≥ 2 {\displaystyle g\geq 2} over number fields K {\displaystyle K} have at most some bounded number N (K , g) {\displaystyle N(K,g)} of K {\displaystyle K} -rational points? Wild problems: problems involving classification of pairs of n × n {\displaystyle n\times n} matrices under simultaneous conjugation. Zariski–Lipman conjecture: for a complex algebraic variety V {\displaystyle V} with coordinate ring R {\displaystyle R} , if the derivations of R {\displaystyle R} are a free module over R {\displaystyle R} , then V {\displaystyle V} is smooth. Zauner's conjecture: do SIC-POVMs exist in all dimensions? Zilber–Pink conjecture that if X {\displaystyle X} is a mixed Shimura variety or semiabelian variety defined over C {\displaystyle \mathbb {C} } , and V ⊆ X {\displaystyle V\subseteq X} is a subvariety, then V {\displaystyle V} contains only finitely many atypical subvarieties. Main article: Group Theory The free Burnside group B (2 , 3) {\displaystyle B(2,3)} is finite; in its Cayley graph, shown here, each of its 27 elements is represented by a vertex. The question of which other groups B (m , n) {\displaystyle B(m,n)} are finite remains open. Andrews–Curtis conjecture: every balanced presentation of the trivial group can be transformed into a trivial presentation by a sequence of Nielsen transformations on relators and conjugations of relators Bounded Burnside problem: for which positive integers m, n is the free Burnside group B(m,n) finite? In particular, is B(2, 5) finite? Guralnick–Thompson conjecture on the composition factors of groups in genus-0 systems[24] Herzog–Schönheim conjecture: if a finite system of left cosets of subgroups of a group G {\displaystyle G} form a partition of G {\displaystyle G} , then the finite indices of said subgroups cannot be distinct. The inverse Galois problem: is every finite group the Galois group of a Galois extension of the rationals? Isomorphism problem of Coxeter groups Are there an infinite number of Leinster groups? Does generalized moonshine exist? Is every finitely presented periodic group finite? Is every group surjunctive? Is every discrete, countable group sofic? Problems in loop theory and quasigroup theory consider generalizations of groups Main article: Representation theory Arthur's conjectures Dade's conjecture relating the numbers of characters of blocks of a finite group to the numbers of characters of blocks of local subgroups. Demazure conjecture on representations of algebraic groups over the integers. Kazhdan–Lusztig conjectures relating the values of the Kazhdan–Lusztig polynomials at 1 with representations of complex semisimple Lie groups and Lie algebras. McKay conjecture: in a group G {\displaystyle G} , the number of irreducible complex characters of degree not divisible by a prime number p {\displaystyle p} is equal to the number of irreducible complex characters of the normalizer of any Sylow p {\displaystyle p} -subgroup within G {\displaystyle G} . Main article: Mathematical analysis The Brennan conjecture: estimating the integral of powers of the moduli of the derivative of conformal maps into the open unit disk, on certain subsets of C {\displaystyle \mathbb {C} } Fuglede's conjecture on whether nonconvex sets in R {\displaystyle \mathbb {R} } and R 2 {\displaystyle \mathbb {R} ^{2}} are spectral if and only if they tile by translation. Goodman's conjecture on the coefficients of multivalued functions Invariant subspace problem – does every bounded operator on a complex Banach space send some non-trivial closed subspace to itself? Kung–Traub conjecture on the optimal order of a multipoint iteration without memory[25] Lehmer's conjecture on the Mahler measure of non-cyclotomic polynomials[26] The mean value problem: given a complex polynomial f {\displaystyle f} of degree d ≥ 2 {\displaystyle d\geq 2} and a complex number z {\displaystyle z} , is there a critical point c {\displaystyle c} of f {\displaystyle f} such that | f (z) − f (c) | ≤ | f ′ (z) | | z − c | {\displaystyle |f(z)-f(c)|\leq |f'(z)||z-c|} ?[27] The Pompeiu problem on the topology of domains for which some nonzero function has integrals that vanish over every congruent copy[28] Sendov's conjecture: if a complex polynomial with degree at least 2 {\displaystyle 2} has all roots in the closed unit disk, then each root is within distance 1 {\displaystyle 1} from some critical point. Vitushkin's conjecture on compact subsets of C {\displaystyle \mathbb {C} } with analytic capacity 0 {\displaystyle 0} What is the exact value of Landau's constants, including Bloch's constant? Regularity of solutions of Vlasov–Maxwell equations Regularity of solutions of Euler equations Convergence of Flint Hills series Regularity of solutions of Vlasov-Maxwell equations Main article: Combinatorics The 1/3–2/3 conjecture – does every finite partially ordered set that is not totally ordered contain two elements x and y such that the probability that x appears before y in a random linear extension is between 1/3 and 2/3?[29] The Dittert conjecture concerning the maximum achieved by a particular function of matrices with real, nonnegative entries satisfying a summation condition Problems in Latin squares – open questions concerning Latin squares The lonely runner conjecture – if k {\displaystyle k} runners with pairwise distinct speeds run round a track of unit length, will every runner be "lonely" (that is, be at least a distance 1 / k {\displaystyle 1/k} from each other runner) at some time?[29] Map folding – various problems in map folding and stamp folding. No-three-in-line problem – how many points can be placed in the n × n {\displaystyle n\times n} grid so that no three of them lie on a line? Rudin's conjecture on the number of squares in finite arithmetic progressions[30] The sunflower conjecture – can the number of k {\displaystyle k} size sets required for the existence of a sunflower of r {\displaystyle r} sets be bounded by an exponential function in k {\displaystyle k} for every fixed r > 2 {\displaystyle r>2} ? Frankl's union-closed sets conjecture – for any family of sets closed under sums there exists an element (of the underlying space) belonging to half or more of the sets[31] Give a combinatorial interpretation of the Kronecker coefficients[32] The values of the Dedekind numbers M (n) {\displaystyle M(n)} for n ≥ 10 {\displaystyle n\geq 10} [33] The values of the Ramsey numbers, particularly R (5 , 5) {\displaystyle R(5,5)} The values of the Van der Waerden numbers Finding a function to model n-step self-avoiding walks[34] Main article: Dynamical system A detail of the Mandelbrot set. It is not known whether the Mandelbrot set is locally connected or not. Arnold–Givental conjecture and Arnold conjecture – relating symplectic geometry to Morse theory. Berry–Tabor conjecture in quantum chaos Banach's problem – is there an ergodic system with simple Lebesgue spectrum?[35] Birkhoff conjecture – if a billiard table is strictly convex and integrable, is its boundary necessarily an ellipse?[36] Collatz conjecture (also known as the 3 n + 1 {\displaystyle 3n+1} conjecture) Eden's conjecture that the supremum of the local Lyapunov dimensions on the global attractor is achieved on a stationary point or an unstable periodic orbit embedded into the attractor. Eremenko's conjecture: every component of the escaping set of an entire transcendental function is unbounded. Fatou conjecture that a quadratic family of maps from the complex plane to itself is hyperbolic for an open dense set of parameters. Furstenberg conjecture – is every invariant and ergodic measure for the × 2 , × 3 {\displaystyle \times 2,\times 3} action on the circle either Lebesgue or atomic? Kaplan–Yorke conjecture on the dimension of an attractor in terms of its Lyapunov exponents Margulis conjecture – measure classification for diagonalizable actions in higher-rank groups. Hilbert–Arnold problem – is there a uniform bound on limit cycles in generic finite-parameter families of vector fields on a sphere? MLC conjecture – is the Mandelbrot set locally connected? Many problems concerning an outer billiard, for example showing that outer billiards relative to almost every convex polygon have unbounded orbits. Quantum unique ergodicity conjecture on the distribution of large-frequency eigenfunctions of the Laplacian on a negatively-curved manifold[37] Rokhlin's multiple mixing problem – are all strongly mixing systems also strongly 3-mixing?[38] Weinstein conjecture – does a regular compact contact type level set of a Hamiltonian on a symplectic manifold carry at least one periodic orbit of the Hamiltonian flow? Does every positive integer generate a juggler sequence terminating at 1? Lyapunov function: Lyapunov's second method for stability – For what classes of ODEs, describing dynamical systems, does Lyapunov's second method, formulated in the classical and canonically generalized forms, define the necessary and sufficient conditions for the (asymptotical) stability of motion? Is every reversible cellular automaton in three or more dimensions locally reversible?[39] Main article: Game theory Main article: Combinatorial game theory Sudoku: How many puzzles have exactly one solution?[40] How many puzzles with exactly one solution are minimal?[40] What is the maximum number of givens for a minimal puzzle?[40] Tic-tac-toe variants: Given the width of a tic-tac-toe board, what is the smallest dimension such that X is guaranteed to have a winning strategy? (See also Hales–Jewett theorem and nd game)[41] Chess: What is the outcome of a perfectly played game of chess? (See also first-move advantage in chess) Go: What is the perfect value of Komi? Are the nim-sequences of all finite octal games eventually periodic? Is the nim-sequence of Grundy's game eventually periodic? Rendezvous problem Main article: Algebraic geometry Abundance conjecture: if the canonical bundle of a projective variety with Kawamata log terminal singularities is nef, then it is semiample. Bass conjecture on the finite generation of certain algebraic K-groups. Bass–Quillen conjecture relating vector bundles over a regular Noetherian ring and over the polynomial ring A [t 1 , … , t n] {\displaystyle A[t_{1},\ldots ,t_{n}]} . Deligne conjecture: any one of numerous named for Pierre Deligne. Deligne's conjecture on Hochschild cohomology about the operadic structure on Hochschild cochain complex. Dixmier conjecture: any endomorphism of a Weyl algebra is an automorphism. Fröberg conjecture on the Hilbert functions of a set of forms. Fujita conjecture regarding the line bundle K M ⊗ L ⊗ m {\displaystyle K_{M}\otimes L^{\otimes m}} constructed from a positive holomorphic line bundle L {\displaystyle L} on a compact complex manifold M {\displaystyle M} and the canonical line bundle K M {\displaystyle K_{M}} of M {\displaystyle M} General elephant problem: do general elephants have at most Du Val singularities? Hartshorne's conjectures[42] In spherical or hyperbolic geometry, must polyhedra with the same volume and Dehn invariant be scissors-congruent?[43] Jacobian conjecture: if a polynomial mapping over a characteristic-0 field has a constant nonzero Jacobian determinant, then it has a regular (i.e. with polynomial components) inverse function. Manin conjecture on the distribution of rational points of bounded height in certain subsets of Fano varieties Maulik–Nekrasov–Okounkov–Pandharipande conjecture on an equivalence between Gromov–Witten theory and Donaldson–Thomas theory[44] Nagata's conjecture on curves, specifically the minimal degree required for a plane algebraic curve to pass through a collection of very general points with prescribed multiplicities. Nagata–Biran conjecture that if X {\displaystyle X} is a smooth algebraic surface and L {\displaystyle L} is an ample line bundle on X {\displaystyle X} of degree d {\displaystyle d} , then for sufficiently large r {\displaystyle r} , the Seshadri constant satisfies ε (p 1 , … , p r ; X , L) = d / r {\displaystyle \varepsilon (p_{1},\ldots ,p_{r};X,L)=d/{\sqrt {r}}} . Nakai conjecture: if a complex algebraic variety has a ring of differential operators generated by its contained derivations, then it must be smooth. Parshin's conjecture: the higher algebraic K-groups of any smooth projective variety defined over a finite field must vanish up to torsion. Section conjecture on splittings of group homomorphisms from fundamental groups of complete smooth curves over finitely-generated fields k {\displaystyle k} to the Galois group of k {\displaystyle k} . Standard conjectures on algebraic cycles Tate conjecture on the connection between algebraic cycles on algebraic varieties and Galois representations on étale cohomology groups. Virasoro conjecture: a certain generating function encoding the Gromov–Witten invariants of a smooth projective variety is fixed by an action of half of the Virasoro algebra. Zariski multiplicity conjecture on the topological equisingularity and equimultiplicity of varieties at singular points[45] Are there infinite sequences of flips possible in dimensions greater than 3? Resolution of singularities in characteristic p {\displaystyle p} Borsuk's problem on upper and lower bounds for the number of smaller-diameter subsets needed to cover a bounded n-dimensional set. The covering problem of Rado: if the union of finitely many axis-parallel squares has unit area, how small can the largest area covered by a disjoint subset of squares be?[46] The Erdős–Oler conjecture: when n {\displaystyle n} is a triangular number, packing n − 1 {\displaystyle n-1} circles in an equilateral triangle requires a triangle of the same size as packing n {\displaystyle n} circles[47] The disk covering problem about finding the smallest real number r (n) {\displaystyle r(n)} such that n {\displaystyle n} disks of radius r (n) {\displaystyle r(n)} can be arranged in such a way as to cover the unit disk. The kissing number problem for dimensions other than 1, 2, 3, 4, 8 and 24[48] Reinhardt's conjecture: the smoothed octagon has the lowest maximum packing density of all centrally-symmetric convex plane sets[49] Sphere packing problems, including the density of the densest packing in dimensions other than 1, 2, 3, 8 and 24, and its asymptotic behavior for high dimensions. Square packing in a square: what is the asymptotic growth rate of wasted space?[50] Ulam's packing conjecture about the identity of the worst-packing convex solid[51] The Tammes problem for numbers of nodes greater than 14 (except 24).[52] Main article: Differential geometry The spherical Bernstein's problem, a generalization of Bernstein's problem Carathéodory conjecture: any convex, closed, and twice-differentiable surface in three-dimensional Euclidean space admits at least two umbilical points. Cartan–Hadamard conjecture: can the classical isoperimetric inequality for subsets of Euclidean space be extended to spaces of nonpositive curvature, known as Cartan–Hadamard manifolds? Chern's conjecture (affine geometry) that the Euler characteristic of a compact affine manifold vanishes. Chern's conjecture for hypersurfaces in spheres, a number of closely related conjectures. Closed curve problem: find (explicit) necessary and sufficient conditions that determine when, given two periodic functions with the same period, the integral curve is closed.[53] The filling area conjecture, that a hemisphere has the minimum area among shortcut-free surfaces in Euclidean space whose boundary forms a closed curve of given length[54] The Hopf conjectures relating the curvature and Euler characteristic of higher-dimensional Riemannian manifolds[55] Osserman conjecture: that every Osserman manifold is either flat or locally isometric to a rank-one symmetric space[56] Yau's conjecture on the first eigenvalue that the first eigenvalue for the Laplace–Beltrami operator on an embedded minimal hypersurface of S n + 1 {\displaystyle S^{n+1}} is n {\displaystyle n} . Main article: Discrete geometry In three dimensions, the kissing number is 12, because 12 non-overlapping unit spheres can be put into contact with a central unit sphere. (Here, the centers of outer spheres form the vertices of a regular icosahedron.) Kissing numbers are only known exactly in dimensions 1, 2, 3, 4, 8 and 24. The big-line-big-clique conjecture on the existence of either many collinear points or many mutually visible points in large planar point sets[57] The Hadwiger conjecture on covering n-dimensional convex bodies with at most 2n smaller copies[58] Solving the happy ending problem for arbitrary n {\displaystyle n} [59] Improving lower and upper bounds for the Heilbronn triangle problem. Kalai's 3d conjecture on the least possible number of faces of centrally symmetric polytopes.[60] The Kobon triangle problem on triangles in line arrangements[61] The Kusner conjecture: at most 2 d {\displaystyle 2d} points can be equidistant in L 1 {\displaystyle L^{1}} spaces[62] The McMullen problem on projectively transforming sets of points into convex position[63] Opaque forest problem on finding opaque sets for various planar shapes How many unit distances can be determined by a set of n points in the Euclidean plane?[64] Finding matching upper and lower bounds for k-sets and halving lines[65] Tripod packing:[66] how many tripods can have their apexes packed into a given cube? Main article: Euclidean geometry The Atiyah conjecture on configurations on the invertibility of a certain n {\displaystyle n} -by- n {\displaystyle n} matrix depending on n {\displaystyle n} points in R 3 {\displaystyle \mathbb {R} ^{3}} [67] Bellman's lost-in-a-forest problem – find the shortest route that is guaranteed to reach the boundary of a given shape, starting at an unknown point of the shape with unknown orientation[68] Borromean rings – are there three unknotted space curves, not all three circles, which cannot be arranged to form this link?[69] Danzer's problem and Conway's dead fly problem – do Danzer sets of bounded density or bounded separation exist?[71] Dissection into orthoschemes – is it possible for simplices of every dimension?[72] Ehrhart's volume conjecture: a convex body K {\displaystyle K} in n {\displaystyle n} dimensions containing a single lattice point in its interior as its center of mass cannot have volume greater than (n + 1) n / n ! {\displaystyle (n+1)^{n}/n!} Falconer's conjecture: sets of Hausdorff dimension greater than d / 2 {\displaystyle d/2} in R d {\displaystyle \mathbb {R} ^{d}} must have a distance set of nonzero Lebesgue measure[73] The values of the Hermite constants for dimensions other than 1–8 and 24 What is the lowest number of faces possible for a polyhedron? Inscribed square problem, also known as Toeplitz' conjecture and the square peg problem – does every Jordan curve have an inscribed square?[74] The Kakeya conjecture – do n {\displaystyle n} -dimensional sets that contain a unit line segment in every direction necessarily have Hausdorff dimension and Minkowski dimension equal to n {\displaystyle n} ?[75] The Kelvin problem on minimum-surface-area partitions of space into equal-volume cells, and the optimality of the Weaire–Phelan structure as a solution to the Kelvin problem[76] Lebesgue's universal covering problem on the minimum-area convex shape in the plane that can cover any shape of diameter one[77] Mahler's conjecture on the product of the volumes of a centrally symmetric convex body and its polar.[78] Moser's worm problem – what is the smallest area of a shape that can cover every unit-length curve in the plane?[79] The moving sofa problem – what is the largest area of a shape that can be maneuvered through a unit-width L-shaped corridor?[80] In parallelohedron: Can every spherical non-convex polyhedron that tiles space by translation have its faces grouped into patches with the same combinatorial structure as a parallelohedron?[81] Does every higher-dimensional tiling by translations of convex polytope tiles have an affine transformation taking it to a Voronoi diagram?[82] Does every convex polyhedron have Rupert's property?[83][84] Shephard's problem (a.k.a. Dürer's conjecture) – does every convex polyhedron have a net, or simple edge-unfolding?[85][86] Is there a non-convex polyhedron without self-intersections with more than seven faces, all of which share an edge with each other? The Thomson problem – what is the minimum energy configuration of n {\displaystyle n} mutually-repelling particles on a unit sphere?[87] Convex uniform 5-polytopes – find and classify the complete set of these shapes[88] Main article: Graph theory Babai's problem: which groups are Babai invariant groups? Brouwer's conjecture on upper bounds for sums of eigenvalues of Laplacians of graphs in terms of their number of edges Does there exist a graph G {\displaystyle G} such that the dominating number γ (G) {\displaystyle \gamma (G)} of G {\displaystyle G} and γ (G) {\displaystyle \gamma (G)} is less than the clique covering number of G {\displaystyle G} ? [89] Graham's pebbling conjecture on the pebbling number of Cartesian products of graphs[90] Meyniel's conjecture that cop number is O (n) {\displaystyle O({\sqrt {n}})} [91] Suppose Alice has a winning strategy for the vertex coloring game on a graph G {\displaystyle G} with k {\displaystyle k} colors. Does she have one for k + 1 {\displaystyle k+1} colors?[92] An instance of the Erdős–Faber–Lovász conjecture: a graph formed from four cliques of four vertices each, any two of which intersect in a single vertex, can be four-colored. The 1-factorization conjecture that if n {\displaystyle n} is odd or even and k ≥ n , n − 1 {\displaystyle k\geq n,n-1} respectively, then a k {\displaystyle k} -regular graph with 2 n {\displaystyle 2n} vertices is 1-factorable. The perfect 1-factorization conjecture that every complete graph on an even number of vertices admits a perfect 1-factorization. Cereceda's conjecture on the diameter of the space of colorings of degenerate graphs[93] The Earth–Moon problem: what is the maximum chromatic number of biplanar graphs?[94] The Erdős–Faber–Lovász conjecture on coloring unions of cliques[95] The graceful tree conjecture that every tree admits a graceful labeling Rosa's conjecture that all triangular cacti are graceful or nearly-graceful The Gyárfás–Sumner conjecture on χ-boundedness of graphs with a forbidden induced tree[96] The Hadwiger conjecture relating coloring to clique minors[97] The Hadwiger–Nelson problem on the chromatic number of unit distance graphs[98] Jaeger's Petersen-coloring conjecture: every bridgeless cubic graph has a cycle-continuous mapping to the Petersen graph[99] The list coloring conjecture: for every graph, the list chromatic index equals the chromatic index[100] The overfull conjecture that a graph with maximum degree Δ (G) ≥ n / 3 {\displaystyle \Delta (G)\geq n/3} is class 2 if and only if it has an overfull subgraph S {\displaystyle S} satisfying Δ (S) = Δ (G) {\displaystyle \Delta (S)=\Delta (G)} . The total coloring conjecture of Behzad and Vizing that the total chromatic number is at most two plus the maximum degree[101] The Albertson conjecture: the crossing number can be lower-bounded by the crossing number of a complete graph with the same chromatic number[102] Conway's thrackle conjecture[103] that thrackles cannot have more edges than vertices The GNRS conjecture on whether minor-closed graph families have ℓ 1 {\displaystyle \ell _{1}} embeddings with bounded distortion[104] Harborth's conjecture: every planar graph can be drawn with integer edge lengths[105] Negami's conjecture on projective-plane embeddings of graphs with planar covers[106] The strong Papadimitriou–Ratajczak conjecture: every polyhedral graph has a convex greedy embedding[107] Turán's brick factory problem – Is there a drawing of any complete bipartite graph with fewer crossings than the number given by Zarankiewicz?[108] Universal point sets of subquadratic size for planar graphs[109] Conway's 99-graph problem: does there exist a strongly regular graph with parameters (99,14,1,2)?[110] Degree diameter problem: given two positive integers d , k {\displaystyle d,k} , what is the largest graph of diameter k {\displaystyle k} such that all vertices have degrees at most d {\displaystyle d} ? Jørgensen's conjecture that every 6-vertex-connected K6-minor-free graph is an apex graph[111] Does a Moore graph with girth 5 and degree 57 exist?[112] Do there exist infinitely many strongly regular geodetic graphs, or any strongly regular geodetic graphs that are not Moore graphs?[113] Barnette's conjecture: every cubic bipartite three-connected planar graph has a Hamiltonian cycle[114] Gilbert–Pollack conjecture on the Steiner ratio of the Euclidean plane that the Steiner ratio is 3 / 2 {\displaystyle {\sqrt {3}}/2} Chvátal's toughness conjecture, that there is a number t such that every t-tough graph is Hamiltonian[115] The cycle double cover conjecture: every bridgeless graph has a family of cycles that includes each edge twice[116] The Erdős–Gyárfás conjecture on cycles with power-of-two lengths in cubic graphs[117] The Erdős–Hajnal conjecture on large cliques or independent sets in graphs with a forbidden induced subgraph[118] The linear arboricity conjecture on decomposing graphs into disjoint unions of paths according to their maximum degree[119] The Lovász conjecture on Hamiltonian paths in symmetric graphs[120] The Oberwolfach problem on which 2-regular graphs have the property that a complete graph on the same number of vertices can be decomposed into edge-disjoint copies of the given graph.[121] What is the largest possible pathwidth of an n-vertex cubic graph?[122] The reconstruction conjecture and new digraph reconstruction conjecture on whether a graph is uniquely determined by its vertex-deleted subgraphs.[123][124] The snake-in-the-box problem: what is the longest possible induced path in an n {\displaystyle n} -dimensional hypercube graph? Sumner's conjecture: does every (2 n − 2) {\displaystyle (2n-2)} -vertex tournament contain as a subgraph every n {\displaystyle n} -vertex oriented tree?[125] Szymanski's conjecture: every permutation on the n {\displaystyle n} -dimensional doubly-directed hypercube graph can be routed with edge-disjoint paths. Tuza's conjecture: if the maximum number of disjoint triangles is ν {\displaystyle \nu } , can all triangles be hit by a set of at most 2 ν {\displaystyle 2\nu } edges?[126] Vizing's conjecture on the domination number of cartesian products of graphs[127] Zarankiewicz problem: how many edges can there be in a bipartite graph on a given number of vertices with no complete bipartite subgraphs of a given size? Are there any graphs on n vertices whose representation requires more than floor(n/2) copies of each letter?[128][129][130][131] Characterise (non-)word-representable planar graphs[128][129][130][131] Characterise word-representable graphs in terms of (induced) forbidden subgraphs.[128][129][130][131] Characterise word-representable near-triangulations containing the complete graph K4 (such a characterisation is known for K4-free planar graphs[132]) Classify graphs with representation number 3, that is, graphs that can be represented using 3 copies of each letter, but cannot be represented using 2 copies of each letter[133] Is it true that out of all bipartite graphs, crown graphs require longest word-representants?[134] Is the line graph of a non-word-representable graph always non-word-representable?[128][129][130][131] Which (hard) problems on graphs can be translated to words representing them and solved on words (efficiently)?[128][129][130][131] The implicit graph conjecture on the existence of implicit representations for slowly-growing hereditary families of graphs[135] Ryser's conjecture relating the maximum matching size and minimum transversal size in hypergraphs The second neighborhood problem: does every oriented graph contain a vertex for which there are at least as many other vertices at distance two as at distance one?[136] Sidorenko's conjecture on homomorphism densities of graphs in graphons Tutte's conjectures: every bridgeless graph has a nowhere-zero 5-flow[137] every Petersen-minor-free bridgeless graph has a nowhere-zero 4-flow[138] Woodall's conjecture that the minimum number of edges in a dicut of a directed graph is equal to the maximum number of disjoint dijoins Main article: Model theory and formal languages The Cherlin–Zilber conjecture: A simple group whose first-order theory is stable in ℵ 0 {\displaystyle \aleph _{0}} is a simple algebraic group over an algebraically closed field. Generalized star height problem: can all regular languages be expressed using generalized regular expressions with limited nesting depths of Kleene stars? For which number fields does Hilbert's tenth problem hold? Kueker's conjecture[139] The main gap conjecture, e.g. for uncountable first order theories, for AECs, and for ℵ 1 {\displaystyle \aleph _{1}} -saturated models of a countable theory.[140] Shelah's categoricity conjecture for L ω 1 , ω {\displaystyle L_{\omega _{1},\omega }} : If a sentence is categorical above the Hanf number then it is categorical in all cardinals above the Hanf number.[140] Shelah's eventual categoricity conjecture: For every cardinal λ {\displaystyle \lambda } there exists a cardinal μ (λ) {\displaystyle \mu (\lambda)} such that if an AEC K with LS(K) ≤ λ {\displaystyle \leq \lambda } is categorical in a cardinal above μ (λ) {\displaystyle \mu (\lambda)} then it is categorical in all cardinals above μ (λ) {\displaystyle \mu (\lambda)} .[140][141] The stable field conjecture: every infinite field with a stable first-order theory is separably closed. The stable forking conjecture for simple theories[142] Tarski's exponential function problem: is the theory of the real numbers with the exponential function decidable? The universality problem for C-free graphs: For which finite sets C of graphs does the class of C-free countable graphs have a universal member under strong embeddings?[143] The universality spectrum problem: Is there a first-order theory whose universality spectrum is minimum?[144] Vaught conjecture: the number of countable models of a first-order complete theory in a countable language is either finite, ℵ 0 {\displaystyle \aleph _{0}} , or 2 ℵ 0 {\displaystyle 2^{\aleph _{0}}} . Assume K is the class of models of a countable first order theory omitting countably many types. If K has a model of cardinality ℵ ω 1 {\displaystyle \aleph _{\omega _{1}}} does it have a model of cardinality continuum?[145] Do the Henson graphs have the finite model property? Does a finitely presented homogeneous structure for a finite relational language have finitely many reducts? Does there exist an o-minimal first order theory with a trans-exponential (rapid growth) function? If the class of atomic models of a complete first order theory is categorical in the ℵ n {\displaystyle \aleph _{n}} , is it categorical in every cardinal?[146][147] Is every infinite, minimal field of characteristic zero algebraically closed? (Here, "minimal" means that every definable subset of the structure is finite or co-finite.) Is the Borel monadic theory of the real order (BMTO) decidable? Is the monadic theory of well-ordering (MTWO) consistently decidable?[148] Is the theory of the field of Laurent series over Z p {\displaystyle \mathbb {Z} _{p}} decidable? of the field of polynomials over C {\displaystyle \mathbb {C} } ? Is there a logic L which satisfies both the Beth property and Δ-interpolation, is compact but does not satisfy the interpolation property?[149] Determine the structure of Keisler's order.[150][151] Main article: Number theory Main article: Probabilistic number theory Does a perfect number exist? Is the Euler–Mascheroni constant irrational? Main article: Additive number theory ... Unsolved problems in mathematics: Are there infinitely many perfect numbers? Is 6 a perfect number because it is the sum of its proper positive divisors, 1, 2 and 3. It is not known how many perfect numbers there are, nor if any of them is odd. Beilinson's conjectures Brocard's problem: are there any integer solutions to n ! + 1 = m 2 {\displaystyle n!+1=m^{2}} other than n = 4 , 5 , 7 {\displaystyle n=4,5,7} ? Büchi's problem on sufficiently large sequences of square numbers with constant second difference. Carmichael's totient function conjecture: do all values of Euler's totient function have multiplicity greater than 1 {\displaystyle 1} ? Casas-Alvero conjecture: if a polynomial of degree d {\displaystyle d} defined over a field K {\displaystyle K} of characteristic 0 {\displaystyle 0} has a factor in common with its first through d − 1 {\displaystyle d-1} -th derivative, then must f {\displaystyle f} be the d {\displaystyle d} -th power of a linear polynomial? Catalan–Dickson conjecture on aliquot sequences: no aliquot sequences are infinite but non-repeating. Erdős–Ulam problem: is there a dense set of points in the plane all at rational distances from one-another? Exponent pair conjecture: for all ε > 0 {\displaystyle \varepsilon >0} , is the pair (ε , 1 / 2 + ε) {\displaystyle (\varepsilon ,1/2+\varepsilon)} an exponent pair? The Gauss circle problem: how far can the number of integer points in a circle centered at the origin be from the area of the circle? Grand Riemann hypothesis: do the nontrivial zeros of all automorphic L-functions lie on the critical line 1 / 2 + i t {\displaystyle 1/2+it} with real t {\displaystyle t} ? Generalized Riemann hypothesis: do the nontrivial zeros of all Dirichlet L-functions lie on the critical line 1 / 2 + i t {\displaystyle 1/2+it} with real t {\displaystyle t} ? Riemann hypothesis: do the nontrivial zeros of the Riemann zeta function lie on the critical line 1 / 2 + i t {\displaystyle 1/2+it} with real t {\displaystyle t} ? Grimm's conjecture: each element of a set of consecutive composite numbers can be assigned a distinct prime number that divides it. Hall's conjecture: for any ε > 0 {\displaystyle \varepsilon >0} , there is some constant c (ε) {\displaystyle c(\varepsilon)} such that either y 2 = x 3 {\displaystyle y^{2}=x^{3}} or | y 2 − x 3 | > c (ε) x 1 / 2 − ε {\displaystyle |y^{2}-x^{3}|>c(\varepsilon)x^{1/2-\varepsilon }} . Hardy–Littlewood zeta function conjectures Hilbert–Pólya conjecture: the nontrivial zeros of the Riemann zeta function correspond to eigenvalues of a self-adjoint operator. Hilbert's eleventh problem: classify quadratic forms over algebraic number fields. Hilbert's ninth problem: find the most general reciprocity law for the norm residues of k {\displaystyle k} -th order in a general algebraic number field, where k {\displaystyle k} is a power of a prime. Hilbert's twelfth problem: extend the Kronecker–Weber theorem on Abelian extensions of Q {\displaystyle \mathbb {Q} } to any base number field. Keating–Snaith conjecture concerning the asymptotics of an integral involving the Riemann zeta function[152] Lehmer's totient problem: if ϕ (n) {\displaystyle \phi (n)} divides n − 1 {\displaystyle n-1} , must n {\displaystyle n} be prime? Leopoldt's conjecture: a p-adic analogue of the regulator of an algebraic number field does not vanish. Lindelöf hypothesis that for all ε > 0 {\displaystyle \varepsilon >0} , ζ (1 / 2 + i t) = o (t ε) {\displaystyle \zeta (1/2+it)=o(t^{\varepsilon })} The density hypothesis for zeroes of the Riemann zeta function Littlewood conjecture: for any two real numbers α , β {\displaystyle \alpha ,\beta } , lim inf n → ∞ n ‖ n α ‖ ‖ n β ‖ = 0 {\displaystyle \liminf _{n\rightarrow \infty }n\Vert n\alpha \Vert \Vert n\beta \Vert =0} , where ‖ x ‖ {\displaystyle \Vert x\Vert } is the distance from x {\displaystyle x} to the nearest integer. Mahler's 3/2 problem that no real number x {\displaystyle x} has the property that the fractional parts of x (3 / 2) n {\displaystyle x(3/2)^{n}} are less than 1 / 2 {\displaystyle 1/2} for all positive integers n {\displaystyle n} . Montgomery's pair correlation conjecture: the normalized pair correlation function between pairs of zeros of the Riemann zeta function is the same as the pair correlation function of random Hermitian matrices. n conjecture: a generalization of the abc conjecture to more than three integers. abc conjecture: for any ε > 0 {\displaystyle \varepsilon >0} , rad (a b c) 1 + ε < c {\displaystyle \operatorname {rad} (abc)^{1+\varepsilon }<c} is true for only finitely many positive a , b , c {\displaystyle a,b,c} such that a + b = c {\displaystyle a+b=c} . Szpiro's conjecture: ... Are there infinitely many primes? ... 1? Is there a covering system with odd distinct moduli?[153] Is π {\displaystyle \pi } a normal number (i.e., is each digit 0–9 equally frequent)?[154] Are all irrational algebraic numbers normal? Is 10 a solitary number? Can a 3×3 magic square be constructed from 9 distinct perfect square numbers?[155] Find the value of the De Bruijn–Newman constant. Main article: Additive number theory See also: Problems involving arithmetic progressions Erdős conjecture on arithmetic progressions that if the sum of the reciprocals of the members of a set of positive integers diverges, then the set contains arbitrarily long arithmetic progressions. Erdős–Turán conjecture on additive bases: if B {\displaystyle B} is an additive basis of order 2 {\displaystyle 2} , then the number of ways that positive integers n {\displaystyle n} can be expressed as the sum of two numbers in B {\displaystyle B} must tend to infinity as n {\displaystyle n} tends to infinity. Gilbreath's conjecture on consecutive applications of the unsigned forward difference operator to the sequence of prime numbers. Goldbach's conjecture: every even natural number greater than 2 {\displaystyle 2} is the sum of two prime numbers. Lander, Parkin, and Selfridge conjecture: if the sum of m {\displaystyle m} k {\displaystyle k} -th powers of positive integers is equal to a different sum of n {\displaystyle n} k {\displaystyle k} -th powers of positive integers, then m + n ≥ k {\displaystyle m+n\geq k} . Lemoine's conjecture: all odd integers greater than 5 {\displaystyle 5} can be represented as the sum of an odd prime number and an even semiprime. Minimum overlap problem of estimating the minimum possible maximum number of times a number appears in the termwise difference of two equally large sets partitioning the set { 1 , … , 2 n } {\displaystyle \{1,\ldots ,2n\}} Pollock's conjectures Does every nonnegative integer appear in Recamán's sequence? Skolem problem: can an algorithm determine if a constant-recursive sequence contains a zero? The values of g(k) and G(k) in Waring's problem Do the Ulam numbers have a positive density? Determine growth rate of rk(N) (see Szemerédi's theorem) Main article: Algebraic number theory Class number problem: are there infinitely many real quadratic number fields with unique factorization? Fontaine–Mazur conjecture: actually numerous conjectures, all proposed by Jean-Marc Fontaine and Barry Mazur. Gan–Gross–Prasad conjecture: a restriction problem in representation theory of real or p-adic Lie groups. Greenberg's conjectures Hermite's problem: is it possible, for any natural number n {\displaystyle n} , to assign a sequence of natural numbers to each real number such that the sequence for x {\displaystyle x} is eventually periodic if and only if x {\displaystyle x} is algebraic of degree n {\displaystyle n} ? Kummer–Vandiver conjecture: primes p {\displaystyle p} do not divide the class number of the maximal real subfield of the p {\displaystyle p} -th cyclotomic field. Lang and Trotter's conjecture on supersingular primes that the number of supersingular primes less than a constant X {\displaystyle X} is within a constant multiple of X / ln ⁡ X {\displaystyle {\sqrt {X}}/\ln {X}} Selberg's 1/4 conjecture: the eigenvalues of the Laplace operator on Maass wave forms of congruence subgroups are at least 1 / 4 {\displaystyle 1/4} . Stark conjectures (including Brumer–Stark conjecture) Characterize all algebraic number fields that have some power basis. Main article: Computational number theory Can integer factorization be done in polynomial time? Further information: Diophantine approximation and transcendental number theory The area of the blue region converges to the Euler–Mascheroni constant, which may or may not be a rational number. Schanuel's conjecture on the transcendence degree of certain field extensions of the rational numbers.[157] In particular: Are π {\displaystyle \pi } and e {\displaystyle e} algebraically independent? Which nontrivial combinations of transcendental numbers (such as e + π , e π , π e , π π , e e {\displaystyle e+\pi ,e\pi ,\pi ^{e},\pi ^{\pi },e^{e}}) are themselves transcendental?[158][159] The four exponentials conjecture: the transcendence of at least one of four exponentials of combinations of irrationals[157] Are Euler's constant γ {\displaystyle \gamma } and Catalan's constant G {\displaystyle G} irrational? Are they transcendental? Is Apéry's constant ζ (3) {\displaystyle \zeta (3)} transcendental?[160][161] Which transcendental numbers are (exponential) periods?[162] How well can non-quadratic irrational numbers be approximated? What is the irrationality measure of specific (suspected) transcendental numbers such as π {\displaystyle \pi } and γ {\displaystyle \gamma } ?[161] Which irrational numbers have simple continued fraction terms whose geometric mean converges to Khinchin's constant?[163] Further information: Diophantine equation Beal's conjecture: for all integral solutions to A x + B y = C z {\displaystyle A^{x}+B^{y}=C^{z}} where x , y , z > 2 {\displaystyle x,y,z>2} , all three numbers A , B , C {\displaystyle A,B,C} must share some prime factor. Congruent number problem (a corollary to Birch and Swinnerton-Dyer conjecture, per Tunnell's theorem): determine precisely what rational numbers are congruent numbers. Erdős–Moser problem: is 1 1 + 2 1 = 3 1 {\displaystyle 1^{1}+2^{1}=3^{1}} the only solution to the Erdős–Moser equation? Erdős–Straus conjecture: for every n ≥ 2 {\displaystyle n\geq 2} , there are positive integers x , y , z {\displaystyle x,y,z} such that 4 / n = 1 / x + 1 / y + 1 / z {\displaystyle 4/n=1/x+1/y+1/z} . Fermat–Catalan conjecture: there are finitely many distinct solutions (a m , b n , c k) {\displaystyle (a^{m},b^{n},c^{k})} to the equation a m + b n = c k {\displaystyle a^{m}+b^{n}=c^{k}} with a , b , c {\displaystyle a,b,c} being positive coprime integers and m , n , k {\displaystyle m,n,k} being positive integers satisfying 1 / m + 1 / n + 1 / k < 1 {\displaystyle 1/m+1/n+1/k<1} . Goormaghtigh conjecture on solutions to (x m − 1) / (x − 1) = (y n − 1) / (y − 1) {\displaystyle (x^{m}-1)/(x-1)=(y^{n}-1)/(y-1)} where x > y > 1 {\displaystyle x>y>1} and m , n > 2 {\displaystyle m,n>2} . The uniqueness conjecture for Markov numbers that every Markov number is the largest number in exactly one normalized solution to the Markov Diophantine equation. Pillai's conjecture: for any A , B , C {\displaystyle A,B,C} , the equation A x m − B y n = C {\displaystyle Ax^{m}-By^{n}=C} has finitely many solutions (x , y , m , n) {\displaystyle (x,y,m,n)} if (m , n) {\displaystyle (m,n)} is not (2 , 2) {\displaystyle (2,2)} . Which integers can be written as the sum of three perfect cubes?[165] Can every integer be written as a sum of four perfect cubes? Main article: Prime numbers Goldbach's conjecture states that all even integers greater than 2 can be written as the sum of two primes. Here this is illustrated for the even integers from 4 to 28. Agoh–Giuga conjecture on the Bernoulli numbers that p {\displaystyle p} is prime if and only if p B p − 1 ≡ − 1 (mod p) {\displaystyle pB_{p-1}\equiv -1{\pmod {p}}} Agrawal's conjecture that given coprime positive integers n {\displaystyle n} and r {\displaystyle r} , if (X − 1) n ≡ X n − 1 (mod n , X r − 1) {\displaystyle (X-1)^{n}\equiv X^{n}-1{\pmod {n,X^{r}-1}}} , then either n {\displaystyle n} is prime or n 2 ≡ 1 (mod r) {\displaystyle n^{2}\equiv 1{\pmod {r}}} Artin's conjecture on primitive roots that if an integer is neither a perfect square nor − 1 {\displaystyle -1} , then it is a primitive root modulo infinitely many prime numbers p {\displaystyle p} Brocard's conjecture: there are always at least 4 {\displaystyle 4} prime numbers between consecutive squares of prime numbers, aside from 2 2 {\displaystyle 2^{2}} and 3 2 {\displaystyle 3^{2}} . Bunyakovsky conjecture: if an integer-coefficient polynomial f {\displaystyle f} has a positive leading coefficient, is irreducible over the integers, and has no common factors over all f (x) {\displaystyle f(x)} where x {\displaystyle x} is a positive integer, then f (x) {\displaystyle f(x)} is prime infinitely often. Catalan's Mersenne conjecture: some Catalan–Mersenne number is composite and thus all Catalan–Mersenne numbers are composite after some point. Dickson's conjecture: for a finite set of linear forms a 1 + b 1 n , … , a k + b k n {\displaystyle a_{1}+b_{1}n,\ldots ,a_{k}+b_{k}n} with each b i ≥ 1 {\displaystyle b_{i}\geq 1} , there are infinitely many n {\displaystyle n} for which all forms are prime, unless there is some congruence condition preventing it. Dubner's conjecture: every even number greater than 4208 {\displaystyle 4208} is the sum of two primes which both have a twin. Elliott–Halberstam conjecture on the distribution of prime numbers in arithmetic progressions. Erdős–Mollin–Walsh conjecture: no three consecutive numbers are all powerful. Feit–Thompson conjecture: for all distinct prime numbers p {\displaystyle p} and q {\displaystyle q} , (p q − 1) / (p − 1) {\displaystyle (p^{q}-1)/(p-1)} does not divide (q p − 1) / (q − 1) {\displaystyle (q^{p}-1)/(q-1)} Fortune's conjecture: that no Fortunate number is composite. The Gaussian moat problem: is it possible to find an infinite sequence of distinct Gaussian prime numbers such that the difference between consecutive numbers in the sequence is bounded? Gillies' conjecture on the distribution of prime divisors of Mersenne numbers. Landau's problems Goldbach conjecture: all even natural numbers greater than 2 {\displaystyle 2} are the sum of two prime numbers. Legendre's conjecture: for every positive integer n {\displaystyle n} , there is a prime between n 2 {\displaystyle n^{2}} and (n + 1) 2 {\displaystyle (n+1)^{2}} . Twin prime conjecture: there are infinitely many twin primes. Are there infinitely many primes of the form n 2 + 1 {\displaystyle n^{2}+1} ? Problems associated to Linnik's theorem New Mersenne conjecture: for any odd natural number p {\displaystyle p} , if any two of the three conditions p = 2 k ± 1 {\displaystyle p=2^{k}\pm 1} or p = 4 k ± 3 {\displaystyle p=4^{k}\pm 3} , 2 p − 1 {\displaystyle 2^{p}-1} is prime, and (2 p + 1) / 3 {\displaystyle (2^{p}+1)/3} is prime are true, then the third condition is true. Polignac's conjecture: for all positive even numbers n {\displaystyle n} , there are infinitely many prime gaps of size n {\displaystyle n} . Schinzel's hypothesis H that for every finite collection { f 1 , … , f k } {\displaystyle \{f_{1},\ldots ,f_{k}\}} of nonconstant irreducible polynomials over the integers with positive leading coefficients, either there are infinitely many positive integers n {\displaystyle n} for which f 1 (n) , … , f k (n) {\displaystyle f_{1}(n),\ldots ,f_{k}(n)} are all primes, or there is some fixed divisor m > 1 {\displaystyle m>1} which, for all n {\displaystyle n} , divides some f i (n) {\displaystyle f_{i}(n)} . Selfridge's conjecture: is 78,557 the lowest Sierpiński number? Does the converse of Wolstenholme's theorem hold for all natural numbers? Are all Euclid numbers square-free? Are all Fermat numbers square-free? Are all Mersenne numbers of prime index square-free? Are there any composite c satisfying 2c − 1 ≡ 1 (mod c2)? Are there any Wall–Sun–Sun primes? Are there any Wieferich primes in base 47? Are there infinitely many balanced primes? Are there infinitely many Carol primes? Are there infinitely many cluster primes? Are there infinitely many cousin primes? Are there infinitely many Cullen primes? Are there infinitely many Euclid primes? Are there infinitely many Fibonacci primes? Are there infinitely many Kummer primes? Are there infinitely many Kynea primes? Are there infinitely many Lucas primes? Are there infinitely many Mersenne primes (Lenstra–Pomerance–Wagstaff conjecture); equivalently, infinitely many even perfect numbers? Are there infinitely many Newman–Shanks–Williams primes? Are there infinitely many palindromic primes to every base? Are there infinitely many Pell primes? Are there infinitely many Pierpont primes? Are there infinitely many prime quadruplets? Are there infinitely many prime triplets? Are there infinitely many regular primes, and if so is their natural density as a subset of all primes e − 1 / 2 {\displaystyle e^{-1/2}} ? Are there infinitely many sexy primes? Are there infinitely many safe and Sophie Germain primes? Are there infinitely many Wagstaff primes? Are there infinitely many Wieferich primes? Are there infinitely many Wilson primes? Are there infinitely many Wolstenholme primes? Are there infinitely many Woodall primes? Can a prime p satisfy 2 p − 1 ≡ 1 (mod p 2) {\displaystyle 2^{p-1}\equiv 1{\pmod {p^{2}}}} and 3 p − 1 ≡ 1 (mod p 2) {\displaystyle 3^{p-1}\equiv 1{\pmod {p^{2}}}} simultaneously?[166] Does every prime number appear in the Euclid–Mullin sequence? What is the smallest Skewes's number? For any given integer a > 0, are there infinitely many primes p such that ap − 1 ≡ 1 (mod p2)?[167] (Specifically, when a = 1, this is the Fibonacci-Wieferich primes, and when a = 2, this is the Pell-Wieferich primes) For any given integer a > 0, are there infinitely many primes p such that ap − 1 ≡ 1 (mod p2)?[167] For any given integer b which is not a perfect power and not of the form −4k4 for integer k, are there infinitely many repunit primes to base b? For any given integers k ≥ 1 , b ≥ 2 , c ≠ 0 {\displaystyle k\geq 1,b\geq 2,c\neq 0} , with gcd(k, c) = 1 and gcd(b, c) = 1, are there infinitely many primes of the form (k × b n + c) / gcd (k + c , b − 1) {\displaystyle (k\times b^{n}+c)/{\gcd(k+c,b-1)}} with integer n ≥ 1? Is every Fermat number 2 2 n + 1 {\displaystyle 2^{2^{n}}+1} composite for n > 4 {\displaystyle n>4} ? Is 509,203 the lowest Riesel number? Main article: Set theory Note: These conjectures are about models of Zermelo-Frankel set theory with choice, and may not be able to be expressed in models of other set theories such as the various constructive set theories or non-wellfounded set theory. (Woodin) Does the generalized continuum hypothesis below a strongly compact cardinal imply the generalized continuum hypothesis everywhere? Does the generalized continuum hypothesis entail ◊ (E c f (λ) λ +) {\displaystyle \diamondsuit (E_{\operatorname {cf} (\lambda)}^{\lambda ^{+}})} for every singular cardinal λ {\displaystyle \lambda } ? Does the generalized continuum hypothesis imply the existence of an ℵ2-Suslin tree? If ℵω is a strong limit cardinal, is 2 ℵ ω < ℵ ω 1 {\displaystyle 2^{\aleph _{\omega }}<\aleph _{\omega _{1}}} ? The (2 ℵ 0 {\displaystyle 2^{\aleph _{0}}}) Reinhardt cardinals: Without assuming the axiom of choice, can a nontrivial elementary embedding V→V exist? Main article: Topology The unknotting problem asks whether there is an efficient algorithm to identify when the shape presented in a knot diagram is actually the unknot. Baum–Connes conjecture: the assembly map is an isomorphism. Berge conjecture that the only knots in the 3-sphere which admit lens space surgeries are Berge knots. Bing–Borsuk conjecture: every n {\displaystyle n} -dimensional homogeneous absolute neighborhood retract is a topological manifold. Borel conjecture: aspherical closed manifolds are determined up to homeomorphism by their fundamental groups. Halperin conjecture on rational Serre spectral sequences of certain fibrations. Hilbert–Smith conjecture: if a locally compact topological group has a continuous, faithful group action on a topological manifold, then the group must be a Lie group. Mazur's conjectures[168] Novikov conjecture on the homotopy invariance of certain polynomials in the Pontryagin classes of a manifold, arising from the fundamental group. Quadrisecants of wild knots: it has been conjectured that wild knots always have infinitely many quadrisecants.[169] Telescope conjecture: the last of Ravenel's conjectures in stable homotopy theory to be resolved.[a] Unknotting problem: can unknots be recognized in polynomial time? Volume conjecture relating quantum invariants of knots to the hyperbolic geometry of their knot complements. Whitehead conjecture: every connected subcomplex of a two-dimensional aspherical CW complex is aspherical. Zeeman conjecture: given a finite contractible two-dimensional CW complex K {\displaystyle K} , is the space K × [0 , 1] {\displaystyle K\times [0,1]} collapsible? Ricci flow, here illustrated with a 2D manifold, was the key tool in Grigori Perelman's solution of the Poincaré conjecture. Mazur's conjecture B (Vesselin Dimitrov, Ziyang Gao, and Philipp Habegger, 2020)[171] Suita conjecture (Qi'an Guan and Xiangyu Zhou, 2015)[172] Torsion conjecture (Loïc Merel, 1996)[173] Carlitz–Wan conjecture (Hendrik Lenstra, 1995)[174] Serre's nonnegativity conjecture (Ofer Gabber, 1995)[175] Kadison–Singer problem (Adam Marcus, Daniel Spielman and Nikhil Srivastava, 2013)[175][176] (and the Feichtinger's conjecture, Anderson's paving conjectures, Weaver's discrepancy theoretic K S r {\displaystyle KS_{r}} and K S r ′ {\displaystyle KS'_{r}} conjectures, Bourgain-Tzafriri conjecture and R ε {\displaystyle R_{\varepsilon }} -conjecture) Ahlfors measure conjecture (Ian Agol, 2004)[177] Gradient conjecture (Krzysztof Kurdyka, Tadeusz Mostowski, Adam Parusiński, 1999)[178] Erdős sumset conjecture (Joel Moreira, Florian Richter, Donald Robertson, 2018)[179] McMullen's g-conjecture on the possible numbers of faces of different dimensions in a simplicial sphere (also Grünbaum conjecture, several conjectures of Kühnel) (Karim Adiprasito, 2018)[180][181] Hirsch conjecture (Francisco Santos Leal, 2010)[182][183] Gessel's lattice path conjecture (Manuel Kauers, Christoph Koutschan, and Doron Zeilberger, 2009)[184] Stanley–Wilf conjecture (Gábor Tardos and Adam Marcus, 2004)[185] (and also the Alon–Friedgut conjecture) Kemnitz's conjecture (Christian Reiher, 2003, Carlos di Fiore, 2003)[186] Cameron–Erdős conjecture (Ben J. Green, 2003, Alexander Sapozhenko, 2003)[187][188] Zimmer's conjecture (Aaron Brown, David Fisher, and Sebastián Hurtado-Salazar, 2017)[189] Painlevé conjecture (Jinxin Xue, 2014)[190][191] Existence of an aperiodic monotile (David Smith, Joseph Samuel Myers, Craig S. Kaplan, Chaim Goodman-Strauss, 2024)[192] The angel problem (Various independent proofs, 2006)[193][194][195][196] Einstein problem (David Smith, Joseph Samuel Myers, Craig S. Kaplan, Chaim Goodman-Strauss, 2024)[197] Maximal rank conjecture (Eric Larson, 2018)[198] Weibel's conjecture (Moritz Kerz, Florian Strunk, and Georg Tamme, 2018)[199] Kelley–Sorensen conjecture (Hendrik Süß? ...)[200][201] Pentagonal tiling (Michaël Rao, 2017)[202] Willmore conjecture (Fernando Codá Marques and André Neves, 2012)[203] Erdős distinct distances problem (Larry Guth, Nets Hawk Katz, 2011)[204] Heterogeneous tiling conjecture (squaring the plane) (Frederick V. Henle and James M. 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